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Isobaric yield ratio difference and Shannon information entropy

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ABSTRACT

The Shannon information entropy theory is used to explain the recently proposed isobaric yield ratio difference (IBD) probe which aims to determine the nuclear symmetry energy. Theoretically, the difference between the Shannon uncertainties carried by isobars in two different reactions (ΔIn_{21}), is found to be equivalent to the difference between the chemical potentials of protons and neutrons of the reactions [the IBD probe, IB-$\Delta(\beta\mu)_{21}$, with β the reverse temperature]. From the viewpoints of Shannon information entropy, the physical meaning of the above chemical potential difference is interpreted by ΔIn_{21} as denoting the nuclear symmetry energy or density difference between neutrons and protons in reactions more concisely than from the statistical ablation–abrasion model.

Nuclear matter with different density range from sub-saturation to supra-saturation can be produced in heavy-ion collisions (HICs). Because of the difficulty to measure the nuclear density and nuclear symmetry energy directly, various probes have been proposed to study the nuclear property based on different models. The results of these probes differ from each other in different extent both theoretically and experimentally [1–7]. The entire process of HICs is dynamical, in which the nuclear matter experience the hot and high density state by violent compressing between projectile and target nuclei, and the dilute states in the process of system expanding. At last, the final residue fragments, which cease to emit particle anymore and are chemically frozen, are measured. Many probes to investigate the nuclear property in HICs are based on the yield of fragments [1,8–16].

The Shannon information entropy, which was put forward by C.E. Shannon, is to measure the uncertainty in a random variable which quantifies the expected value of the information contained in a message [17]. The Shannon entropy tells the average unpredictability in a random variable, which is equivalent to its information content, and provides a constructive criterion for setting up probability distributions on the basis of partial knowledge, and leads to a type of statistical inference called as the maximum-entropy estimate [18]. In the information communication, the Shannon entropy provides an absolute limit on the best possible lossless encoding or compression of any communication, assuming that the communication may be represented as a sequence of independent and identically distributed random variables [19]. The original information entropy measures the "amount of information" which is contained in messages sent along a transmission line, and it has been further applied in a wide variety of problems in economics, engineering, and many other fields [20]. The information entropy method, which is defined as the system entropy (entropy in whole reaction system) and eventropy (entropy in event space), was introduced in the study of the multiparticle production in the high energy hadron collisions, and verified that the information theory is a good tool to measure the chaoticity in the hadron decaying branching process [21]. By defining the multiplicity information entropy, the information entropy was introduced to study the nuclear disassembly and the liquid–gas phase transition in HICs for the first time by Y.G. Ma [22].

Considering the dynamical process of HICs as an information communication process, the nuclear matter in HICs can be studied by using the Shannon entropy theory. Carrying the information of the reaction, the measured fragments in HICs compose a system and can be defined as independent samples. The probability of one specific isotope is denoted by its production cross section (σ). Following the definition of information entropy, in a system which has multi events $S = \{E_1, E_2, \cdots, E_n\}$ and the corresponding probabilities $\{p_1, p_2, \cdots, p_n\}$, the information uncertainty of a

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certain event E_i (or the information uncertainty that the event E_i contains) is defined as,

$$In(E_i) = -\ln p_i, \tag{1}$$

with the unit of $In(E_i)$ in *nats*. If the probability of the event is non-uniform, the information entropy of the system is defined as [17,18,21,22],

$$H(S) = -\sum_{i=1}^{n} p_i \ln p_i. \tag{2}$$

It is explained that the information entropy and information uncertainty can be used interchangeably [18]. In some applications, $In(E_i)$ is also named as information entropy of one event. To differ the concept of the previously defined information entropy in Refs. [21,22], $In(E_i)$ is called as the *information uncertainty* and $H(S)$ as the *information entropy*.

We aim at explaining the recently proposed probe to the nuclear density in HICs, which is called as IBD (denoting "isobaric yield ratio difference"), based on the information uncertainty method. In thermal equilibrium models, the yield of a fragment is mainly decided by three aspects – the property of the colliding source, the free energy of fragment, and the temperature of the system. Taking the grand canonical ensemble theory as an example, the yield of a fragment $[\sigma(A, I)]$ is given by [23,24],

$$\sigma(A, I) = CA^{\tau} \exp\{\beta[-F(A, I) + \mu_n N + \mu_p Z]\}, \tag{3}$$

where A, N (Z) and $I(\equiv N - Z)$ are the mass, neutron (proton) numbers, and neutron-excess, respectively; C and τ are constants depending on the reaction system; β is the reverse temperature; μ_n (μ_p) is the chemical potential of neutrons (protons), which depends on nuclear density and temperature; $F(A, I)$ is the free energy of the fragment depending both on the density and the temperature. Inserting Eq. (3) into Eq. (1), the information uncertainty of the fragment (A, I) can be obtained,

$$In^*(A, I) = -\ln\sigma(A, I), \tag{4}$$

where a star is used in Eq. (4) since in Eq. (1) $\sum_{i=1}^{m} p_i = 1$ (m denotes numbers of fragment species) is physically required [21,22]. Summing up $\sigma(A, I)$ of all the fragments $[\sigma_{Ft} \equiv \sum_{i=1}^{m} \sigma_i(A, I)]$, one has $\sum_{i=1}^{m} \sigma(A, I)/\sigma_{Ft} = 1$. From Eq. (4), the information uncertainties of all the fragments satisfy $\sum_{i=1}^{m} p_i = 1$, which results in,

$$In(A, I) = -\ln[\sigma(A, I)/\sigma_{Ft}] = -\ln\sigma(A, I) + const. \tag{5}$$

In isobaric yield ratio, many terms in $\sigma(A, I)$ are canceled out and useful information can be obtained [14,15,25–32]. The system dependent parameters in $\sigma(A, I)$, such as CA^{τ} and *const*., disappear in the difference between the $In(A, I)$ of isobars, which becomes,

$$\begin{aligned}\Delta In &= In(A, I+2) - In(A, I)\\ &= \beta[F(A, I+2) - F(A, I) + \mu_p - \mu_n],\end{aligned} \tag{6}$$

where the information about the free energy $[F(A, I+2) - F(A, I)]$ and the chemical potential difference $(\mu_p - \mu_n)$ are retained in ΔIn. All of the quantities $[F(A, I+2) - F(A, I)]$, $(\mu_p - \mu_n)$ and β are difficult to be directly measured. Considering two reactions of similar experimental setups (reaction 1 is the symmetric reference reaction, and reaction 2 is a neutron-rich system), the temperature as well as the free energy of one fragment in the two reactions can be assumed as the same [16]. The difference of ΔIn between the two reactions is defined as,

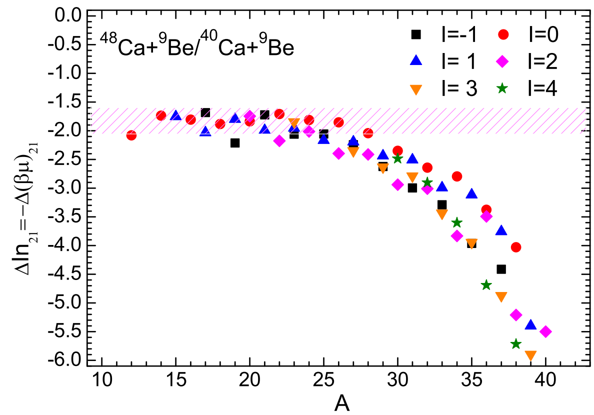

Fig. 1. (Color online.) The entropy uncertainty ΔIn_{21} and the results of IBD between the 140*A* MeV ^{48}Ca + ^{9}Be and ^{40}Ca + ^{9}Be reactions [37]. The shadow area denotes the range of -1.85 ± 0.25.

$$\begin{aligned}\Delta In_{21} &= \Delta In_2 - \Delta In_1 = \beta[(\mu_{n1} - \mu_{n2}) - (\mu_{p1} - \mu_{p2})]\\ &= \Delta(\beta\mu)_{12}.\end{aligned} \tag{7}$$

The free energy terms of the isobars are eliminated. In the definition of the IBD probe [denoted as the IBD-$\Delta(\beta\mu)_{21}$], which is [29, 30],

$$\Delta(\beta\mu)_{21} = \beta[(\mu_{n2} - \mu_{n1}) - (\mu_{p2} - \mu_{p1})]. \tag{8}$$

The quantity ΔIn_{21} equals to the negative value of the IBD-$\Delta(\beta\mu)_{21}$, i.e., $\Delta In_{21} = -\Delta(\beta\mu)_{21}$. In the isoscaling methods, $\mu_n(\mu_p)$ is related to the neutron (proton) density $\ln\rho_n(\ln\rho_p)$. The quantity $(\mu_n - \mu_p)$ reflects the difference between the neutrons and protons densities $(\Delta\ln\rho \equiv \ln\rho_n - \ln\rho_p)$ in one colliding source, the quantity $\Delta(\beta\mu)_{21}$ reflects the difference between $\Delta\ln\rho$ of the two sources [31,33,34]. The Shannon information uncertainty difference ΔIn_{21} and the IBD-$\Delta(\beta\mu)_{21}$ can be obtained directly from the cross sections of isobars and no fitting is needed, which make them possible to directly probe to the chemical potentials and nuclear density comparing to the isoscaling methods [32]. In the isoscaling methods, α (β) can be easily related to the nuclear symmetry energy (C_{sym}/T) since they reflect the value of $\Delta(\beta\mu_{n21})$ $(\Delta(\beta\mu_{p21}))$ [2,8,14,35,36]. While the ΔIn_{21} and $\Delta(\beta\mu)_{21}$ are related to μ_n and μ_p at the same time, which make it difficult to be related C_{sym}/T. To make correlations between ΔIn_{21} (or $\Delta(\beta\mu)_{12}$) and C_{sym}/T, more approximations are needed.

Now we try to use Eq. (7) to estimate the information uncertainty in HICs. The yields of the fragments produced in the 140*A* MeV ^{40,48}Ca + ^{9}Be and the ^{58,64}Ni + ^{9}Be reactions were measured by M. Mocko et al. at the National Superconducting Cyclotron Laboratory (NSCL) in Michigan State University [37]. The $\Delta(\beta\mu)_{21}$ in these reactions has been studied by using the IBD method, which found that the results of IBD agree with those of the isoscaling method [29]. In this letter, the yields of fragments in the ^{40,48}Ca + ^{9}Be, and the ^{58}Ni/^{40}Ca + ^{9}Be reactions will be studied to show the information uncertainty carried by the measured fragments.

First, ΔIn_{21} between the isobars in the 140*A* MeV ^{48}Ca + ^{9}Be and ^{40}Ca+^{9}Be (denoted as ^{48}Ca/^{40}Ca) reactions is plotted in Fig. 1. For fragments in each of the I-chain, ΔIn_{21} has the distribution of a plateau part plus a decreasing part with the increasing A of fragments. The plateaus of ΔIn_{21} (in the $A \le 25$ isobars) for the different I-chains are relative consistent within the range of -1.85 ± 0.25. The value of ΔIn_{21} decreases fast with A when $A > 28$.

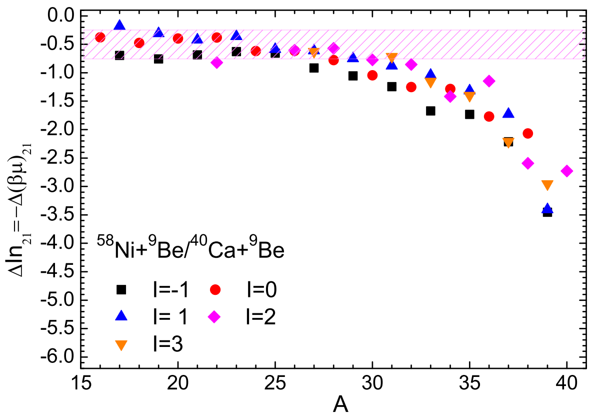

Fig. 2. (Color online.) The entropy uncertainty ΔIn_{21} and the results of IBD between the 140A MeV ^{58}Ni + ^{9}Be and ^{40}Ca + ^{9}Be reactions [37]. The shadow area denotes the range of -0.5 ± 0.25.

Second, ΔIn_{21} between the isobars in the 140A MeV ^{58}Ni + ^{9}Be and ^{40}Ca + ^{9}Be (denoted as ^{58}Ni/^{40}Ca) reactions is plotted in Fig. 2. It is noted that CA^{τ} and *const.*, which depend on the reaction systems in isobars, are canceled out in Eqs. (6) and (7). The isobars in reactions, besides the reaction of isotopic projectiles, can be analyzed to obtain the information uncertainty. In the ^{58}Ni/^{40}Ca reactions, a similar trend of ΔIn_{21} as shown in the ^{48}Ca/^{40}Ca reactions is found. The values of plateaus for the ΔIn_{21} in the ^{58}Ni/^{40}Ca reactions are larger than those in the ^{48}Ca/^{40}Ca reactions, and the difference of the plateau values between the different I-chains is slightly larger compared to those in the ^{48}Ca/^{40}Ca reactions.

To explain the phenomena shown in the IBD-$\Delta(\beta\mu)_{21}$ [29,30], the ideas of the geometric model, for example the statistical abrasion ablation (SAA) model, were introduced [11,38–42]. The explanation is briefly summarized as follows. In the SAA model, the yield of fragment to some extent is mainly determined by ρ_n, ρ_p, and nucleon–nucleon cross sections (σ_{NN}). Labeling the difference between the proton densities in the two reactions as $\Delta \ln \rho_{p21} = \ln \rho_{p2} - \ln \rho_{p1}$, and that between the neutrons as $\Delta \ln \rho_{n21} = \ln \rho_{n2} - \ln \rho_{n1}$, the IBD-$\Delta(\beta\mu)_{21}$ is found to denote the neutron-and-proton-density difference $\Delta \ln \rho_{np21} (\equiv \Delta \ln \rho_{n21} - \Delta \ln \rho_{p21})$ between the two reactions [29,30].[1] In the ^{48}Ca/^{40}Ca isotopic reactions, assuming that ρ_p are the same, the value of $\Delta(\beta\mu)_{21}$ denotes $\Delta \ln \rho_{n21}$ between the reactions. The plateaus denote the regions in the two reactions (which correspond to the central collisions) where ρ_n and ρ_p vary slowly and $\Delta \ln \rho_{np21}$ changes small, and the heights of the plateaus denote the value of $\Delta \ln \rho_{np21}$. The increasing $\Delta(\beta\mu)_{21}$ with A of fragment is explained as the enlarged $\Delta \ln \rho_{np21}$ in the surface regions of the projectiles or in the peripheral reactions, which was also suggested as the neutron-skin effects in reactions of neutron-rich projectile nucleus [43,44]. The assumptions based on the SAA model explain the result of $\Delta(\beta\mu)_{21}$ in a complicated way. From the point view of the information theory, the physical meaning of $\Delta(\beta\mu)_{21}$ can be interpreted more clearly. Though information besides $\mu_n(\mu_p)$ is carried in the fragment yield, the free energy of fragment and the system dependence of fragment yield are canceled out in the isobaric yield ratio. Noting that μ_n and μ_p are determined by the density of the source, from Eq. (6), the values of ΔIn_{21} reflects the quantity of $\Delta \ln \rho_{np21}$. Since $\Delta In_{21} = -\Delta(\beta\mu)_{21}$, the explanation of the IBD results also suits for ΔIn_{21}. In Figs. 1 and 2, the heights of the ΔIn_{21} plateaus in the ^{48}Ca/^{40}Ca reactions are in the range of -1.85 ± 0.25, while they become to -0.5 ± 0.25 in the ^{58}Ni/^{40}Ca reactions. The values of ΔIn_{21} for fragments with $30 < A < 40$ in the ^{48}Ca/^{40}Ca reactions are also smaller than those in the ^{58}Ni/^{40}Ca reactions. It is concluded that ΔIn_{21} and $\Delta(\beta\mu)_{21}$ both denote the $\Delta \ln \rho_{np21}$ between the reactions. But the exact relationship between ΔIn_{21} and $\Delta \ln \rho_{np21}$ is not established, which makes it difficult to know the value of $\Delta \ln \rho_{np21}$ at the chemical freeze-out stage. Besides, the decay effect is presently not considered in Eq. (3), thus ΔIn_{21} will potentially be influenced by the decay processes [30].

Actually, the concepts of temperature and chemical potentials are for the thermal equilibrium state defined in the thermodynamics. As has been noted, the HICs are highly dynamical process involving fast change of nuclear densities, and no global thermal equilibrium but only local equilibrium can be achieved. The IBD probe is for the static equilibrium stage of the reaction. In the dynamical process, the IBD probe and the isoscaling method face great challenges. But the Shannon information entropy uncertainty ΔIn_{21} can also be used to check the results in the dynamical process. In transport models, such as the quantum molecular dynamical (QMD) model [45–49], the antisymmetric molecular dynamical (AMD) model [14,50–52], or the constrained molecular dynamics (CoMD) [53,54] model, the yields of fragments can be well produced. The concept of the information uncertainty can be used to study the evolution of nuclear matter in the reaction system, in which the time of chemical freeze-out in thermodynamics corresponds to the final state. We will study the information evolution with the reaction time in future in some of the dynamical models.

It is stressed by Jaynes that, in the problem of prediction, the maximization of entropy is not an application of a law of physics, but merely a method of reasoning which ensures that no unconscious arbitrary assumptions have been introduced [18]. Though no exact value of $\Delta \ln \rho_{np21}$ is obtained from ΔIn_{21} or $\Delta(\beta\mu)_{21}$, the probes ΔIn_{21} or $\Delta(\beta\mu)_{21}$ point out one possible way to further understand the nuclear density and nuclear symmetry energy in HICs, which calls for further investigation.

To summarize, the recently proposed IBD probe, which aims to study the density difference between neutrons and protons in HICs, is explained in a simple manner in the framework of the information entropy theory. The difference between the information uncertainty carried by the isobars in two reactions, ΔIn_{21}, is found to has the same value as the negative IBD-$\Delta(\beta\mu)_{21}$. By avoiding the theoretical complexity in describing the fragment production, ΔIn_{21} explains the physical meaning of IBD-$\Delta(\beta\mu)_{21}$ in an explicit manner. It is proposed that both ΔIn_{21} and IBD-$\Delta(\beta\mu)_{21}$ denote the difference of neutrons and protons densities between the reactions. But from the view of the information evolution in HICs, the results of ΔIn_{21} can provide new results of the systematic evolution of the reaction system. Since the IBD result in the thermodynamical model is for the equilibrium state of the reaction, it is proposed to study the evolution of ΔIn_{21} in transport models, which is useful to understand the evolutions of nuclear density and nuclear symmetry energy in HICs.

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[1] In Refs. [29–31], the difference between ρ_n and ρ_p was used to discuss the IBD results, but it was changed to use the difference between $\ln \rho_n$ and $\ln \rho_p$ to discuss the results in Ref. [34].

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