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Gain of harmonic generation in high gain free electron laser*

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Abstract In a planar undulator employed free electron laser (FEL), each harmonic radiation starts from linear amplification and ends with nonlinear harmonic interactions of the lower nonlinear harmonics and the fundamental radiation. In this paper, we investigate the harmonic generation based on the dispersion relation driven from the coupled Maxwell-Vlasov equations, taking into account the effects due to energy spread, emittance, betatron oscillation of electron beam as well as diffraction guiding of the radiation field. A 3D universal scaling function for gain of the linear harmonic generation and a 1D universal scaling function for gain of the nonlinear harmonic generation are presented, which promise rapid computation in FEL design and optimization. The analytical approaches have been validated by 3D simulation results in large range.

Key words harmonic, FEL, gain length

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1 Introduction

Free electron lasers (FELs) are devices that use the relativistic electron beams passing through a transverse periodic magnetic field in order to generate coherent electromagnetic radiation ranging from the infrared to hard X-ray region[1]. Whether in the case of a FEL oscillator or a single-pass FEL, the gain length of the amplified radiation must be minimized to achieve good FEL performance. Accordingly, three dimensional (3D) FEL codes such as GENESIS1.3[2] and TDA3D[3] have been developed to simulate FEL performance. However, these codes require long CPU running time on fast computers in FEL design and optimization, making it an effort-consuming process. Thus, the analytical estimate of FEL gain, an interesting issue all through, has been studied by many authors[1, 4—7]. Further study clarified the effects of energy spread, emittance, and the focusing[8] of the electron beam, and the diffraction and guiding[9] of the radiation field in succession. The next important step was the first 3D equations derived by Kim[10] to deal with these effects simultaneously, but without providing a solution. Using a variational technique introduced by Xie[11], a universal scaling function for FEL gain which is the first approximate solution of the 3D equation was given by Yu[12]. In Yu's solution, they assumed a "water-bag" model for unperturbed electron distribution in transverse phase space and solved the dispersion relation[13] from coupled Maxwell-Vlasov equations for the fundamental mode. Later, Hafizi[14] considered a sheet beam of Gaussian model, also for the fundamental mode. With a novel approach in handling the coupled Maxwell-Vlasov equations, Chin[15] derived an equation fitting for any initial beam distribution and obtained another approximate solution for the fundamental mode. Moreover, Xie[16] provided a collection of formulas relating FEL performance without derivation, which allow quick evaluation of FEL design and optimization for fundamental mode in multidimensional parameters space, and by which LCLS FEL optimization has been carried out[17].

In recent years, people are increasingly interested by high gain and short wavelength FEL. Self amplified spontaneous emission[18] and high gain harmonic generation[19] FEL are two leading candidates for approaching hard X-ray region. In comparison with the

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substantive cost in attaining a high energy electron beam to achieve short wavelength region, harmonic radiation in FEL holds great promise to reach shorter radiation wavelengths or to relax some of the stringent requirements on electron beam quality. Therefore, investigation of the harmonic radiation in FEL is on its way. On the one hand, simulation codes such as GINGER[20], MEDUSA[21] and TDA-H[22] are developed or modified to simulate harmonic generation. On the other hand, the analytical calculations of the gain on the higher order odd harmonics[23], the even harmonics and the off axis[24] were ongoing. Bonifacio[25] gives the 1D model to illuminate the linear harmonic generation due to initial smooth distribution of electron beam and the nonlinear harmonic interaction due to strong bunching in the fundamental. Dattoli[26] gives a set of expressions of harmonic radiation with the limitation of the 1D treatment. Kim and Huang[27] presented a 3D analysis of harmonic radiation, in which the matrix formulation of Xie[28] is employed to solve the dispersion relation on the basis of the coupled Maxwell-Klimontovich equations, and their results have been validated by the numerical simulations[29] and experimental measurements[30], however, they mainly concentrate on the nonlinear harmonic interactions.

Here, we shall extend Yu's work in FEL gain[12] for the fundamental wavelength to the harmonics, and give a more universal scaling function for FEL gain. A planar undulator is assumed throughout the discussion. Since in a planar undulator employed high gain FEL, the electron beam motion causes the odd harmonics to be the most significant in the forward direction, we only consider the odd harmonics in this discussion. In this paper, we first describe the interaction between the electron beam and radiation field in Section 2 by coupled Maxwell-Vlasov equations. In Section 3, based on the dispersion relation driven from the coupled equations, the scaling function for the linear harmonic gain is derived. Thirdly in Section 4, we give a simple discussion on the nonlinear harmonic generation. Then, for the FEL parameters in different spectral regions, we check our analytical approach with the results simulated by 3D FEL code TDA-H in Section 5. Finally, we present our conclusions in Section 6.

2 Coupled Maxwell-Vlasov equations

Suppose a planar undulator with a sinusoidal magnetic field in the y direction. Considering a relativistic electron beam with average energy $\gamma_0 mc^2$ entering the undulator in the z direction, one observes the transverse wiggling motion, together with the "figure of eight" longitudinal phase oscillations in the resonant frame. This trajectory can give rise to harmonic radiation. We denote the fundamental radiation wavelength by $\lambda_{\rm s}$ and the period length of the undulator magnet by $\lambda_{\rm w}$ The corresponding wave numbers are $k_{\rm s}=2\pi/\lambda_{\rm s}$ and $k_{\rm w}=2\pi/\lambda_{\rm w}$. Then, the vector potential of planar undulator has the form

$$\boldsymbol{A}_{\rm w}=A_{\rm w}\cos(k_{\rm w}z)\hat{x} \tag{1}$$

and the longitudinal velocity of the electron can be approximated by

$$v_{\parallel}\approx c\left(1-\frac{1+K^2\cos^2(k_{\rm w}z)}{2\gamma^2}\right), \tag{2}$$

where $K\equiv eA_{\rm w}/mc$ is the undulator magnetic parameter. Now we represent the electric field in the form

$$E=\frac{1}{2}\sum_{n=1}^{\infty}E_n(r,z,t)\exp[{\rm i}n(k_{\rm s}z-\omega_{\rm s}t)]+c.c., \tag{3}$$

where $r=(x,y)$ represents the transverse coordinates, n is the odd number and $E_n(r,z,t)$ is the slowly varying envelope function. The radiation electric field $E_n(r,z,t)$ satisfies the Maxwell's equation as follows, in MKS units,

$$\frac{1}{2}\left(\nabla_{\perp}^2+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\times \left(\sum_{n=1}^{\infty}\frac{1}{{\rm i}n\omega_{\rm s}}E_n{\rm e}^{{\rm i}n(k_{\rm s}z-\omega_{\rm s}t)}\right)+c.c.=-\mu_0 J_x\,, \tag{4}$$

where μ_0 is the permeability of free space, and $\nabla_{\perp}^2$ is the transverse Laplacian. Denoting the energy and position of the j-th electron by $\gamma_j mc^2$ and $(r_j,\,z_j)$, the transverse current density for a beam of N electrons is given by

$$J_x=ecK\cos(k_{\rm w}z)\sum_{j=1}^{N}\frac{1}{\gamma_j}\delta[r-r_j(t)][z-z_j(t)]. \tag{5}$$

The wave equation is simplified by using the paraxial approximation,

$$\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\approx 2{\rm i}nk_{\rm s}\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right). \tag{6}$$

If we approximate $\gamma_0=\gamma_j$ in the transverse velocity of the beam and drop the exponential terms, then

$$\left(\frac{1}{2{\rm i}nk_{\rm s}}\nabla_{\perp}^2+\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)E_n=-\mu_0 cJ_x{\rm e}^{-{\rm i}n(k_{\rm s}z-\omega_{\rm s}t)}= -\mu_0 ec^2\frac{K}{\gamma_0}\sum_{j=1}^{N}\cos(k_{\rm w}z){\rm e}^{-{\rm i}n(k_{\rm s}z-\omega_{\rm s}t)}\times \delta[r-r_j(t)][z-z_j(t)]. \tag{7}$$

We average Eq. (7) over the undulator period $\lambda_{\rm w}$ with the help of the Bessel function expansion

$${\rm e}^{-{\rm i}n(k_{\rm s}z-\omega_{\rm s}t)}\cos(k_{\rm w}z)\cong\frac{1}{2}{\rm e}^{-{\rm i}\theta}[JJ]_n\,. \tag{8}$$

Here, $\theta = k_{\mathrm{w}}z + k_{\mathrm{s}}z - \omega_{\mathrm{s}}t + \xi\sin(2k_{\mathrm{w}}z)$ describes the FEL bunching action, $[JJ]_n$ is the difference of the Bessel functions defined as $[JJ]_n = (-1)^{(n-1)/2}[J_{(n-1)/2}(n\xi) - J_{(n+1)/2}(n\xi)]$, where $\xi = K^2/(4+2K^2)$. Then, we have the radiation electric field equation from Eq. (7)

$$\left(\frac{1}{2\mathrm{i}nk_{\mathrm{s}}}\nabla_\perp^2 + \frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)E_n = \frac{\mu_0 ec^2K[JJ]_n}{2\gamma_0}\sum_{j=1}^{N}\mathrm{e}^{-\mathrm{i}\theta_j}\delta(r-r_j)\delta(z-z_j)\,. \quad (9)$$

Furthermore, we let n_0 be the peak density and $n_0 f(z,t,\gamma,r)$ be the distribution function such that $n(z,t,r) = \int n_0 f(z,t,\gamma,r)\mathrm{d}\gamma$ be the particle density. If we average the right hand side of Eq. (9) over a small volume

$$\left(\frac{1}{2\mathrm{i}nk_{\mathrm{s}}}\nabla_\perp^2 + \frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)E_n = \frac{\mu_0 n_0 ec^2K[JJ]_n}{2\gamma_0} < \mathrm{e}^{-\mathrm{i}\theta} > \int f\mathrm{d}\gamma\,. \quad (10)$$

Now it is useful to introduce the dimensionless variables

$$\begin{aligned} \tau &= k_{\mathrm{w}}z, \\ x &= r\sqrt{2k_{\mathrm{s}}k_{\mathrm{w}}}, \\ p &= \mathrm{d}x/\mathrm{d}\tau, \\ k &= k_\beta/k_{\mathrm{w}}, \\ f &= \sum_{n=0}^{\infty}F_n\mathrm{e}^{\mathrm{i}n\theta} + c.c. \end{aligned}$$

In these definitions, F_0 represents the smooth distribution in the absence of the radiation field, and other components F_n, near $\mathrm{e}^{\mathrm{i}n\theta}$, represent the corresponding electron beam bunching which will contribute to the growth of the n-th harmonic radiation significantly. Then, the evolution of the Vlasov distribution function f can be governed by the continuity equation

$$\left(\frac{\partial}{\partial\tau} + \theta'\frac{\partial}{\partial\theta} + \gamma'\frac{\partial}{\partial\gamma} + x'\frac{\partial}{\partial x} + p'\frac{\partial}{\partial p}\right)f = 0\,. \quad (11)$$

Thus, if we arrange the slow and fast components of the beam distribution function properly and neglect some oscillating terms, the coupled Maxwell-Vlasov equations can now be written as

$$\left(\frac{\partial}{\partial\tau} + \frac{\partial}{\partial\theta} - \frac{\mathrm{i}}{n}\nabla_\perp^2\right)E_n = \frac{D_{1n}}{\gamma_0}\int\mathrm{d}\gamma\int F_n\mathrm{d}^2p\,, \quad (12)$$

$$\left(\frac{\partial}{\partial\tau} + \theta'\frac{\partial}{\partial\theta} + p\frac{\partial}{\partial x} - k^2x\frac{\partial}{\partial p}\right)\left(\sum_{n=0}^{\infty}F_n\mathrm{e}^{\mathrm{i}n\theta}\right) = \left(\sum_{n=1}^{\infty}\frac{D_{2n}}{\gamma}E_n\mathrm{e}^{\mathrm{i}n\theta}\right)\frac{\partial}{\partial\gamma}\left(\sum_{n=0}^{\infty}F_n\mathrm{e}^{\mathrm{i}n\theta}\right), \quad (13)$$

with $D_{1n} = n_0\mu_0 ec^2K[JJ]_n/2k_{\mathrm{w}}$, $D_{2n} = eK[JJ]_n/4k_{\mathrm{w}}mc^2$, and

$$\begin{aligned} \theta' &\approx 2\frac{\gamma-\gamma_0}{\gamma_0} - \frac{1}{4}(p^2 + k^2x^2), \\ \gamma' &= \sum_{n=1}^{\infty}\frac{D_{2n}}{\gamma}E_n\mathrm{e}^{\mathrm{i}n\theta} + c.c. \end{aligned} \quad (14)$$

According to Eq. (12), the gain of the n-th harmonic radiation is determined by the term F_n, therefore, we separate the different order of $\mathrm{e}^{\mathrm{i}\theta}$ in Eq. (13) and have

$$\left[\frac{\partial}{\partial\tau} + \theta'\left(\frac{\partial}{\partial\theta} + \mathrm{i}n\right) + p\frac{\partial}{\partial x} - k^2x\frac{\partial}{\partial p}\right]F_n = \sum_{u=1}^{u+v=n}\frac{D_{2u}}{\gamma_0}E_u\frac{\partial F_v}{\partial\gamma}\,. \quad (15)$$

Now we introduce Fourier transform over θ,

$$E_n(\tau,x,q) = \int_{-\infty}^{+\infty}\mathrm{d}\theta\mathrm{e}^{-\mathrm{i}q\theta}E_n(\tau,x,\theta)\,, \quad (16)$$

$$F_n(\tau,x,q,\gamma) = \int_{-\infty}^{+\infty}\mathrm{d}\theta\mathrm{e}^{-\mathrm{i}q\theta}F_n(\tau,x,\theta,\gamma) \quad (17)$$

and Laplacian transform over τ,

$$E_n(\Omega,x,q) = \int_0^{\infty}\mathrm{d}\tau\mathrm{e}^{\mathrm{i}\Omega\tau}E_n(\tau,x,q)\,, \quad (18)$$

$$F_n(\Omega,x,q,\gamma) = \int_0^{\infty}\mathrm{d}\tau\mathrm{e}^{\mathrm{i}\Omega\tau}F_n(\tau,x,q,\gamma)\,. \quad (19)$$

For simplicity, we use the same symbol E_n and F_n for the functions and their transforms. Then the Maxwell-Vlasov equation including the 3D effects can be reduced as the following

$$(-\mathrm{i}\Omega + \mathrm{i}q - \frac{\mathrm{i}}{n}\nabla_\perp^2)E_n = \frac{D_{1n}}{\gamma_0}\int\mathrm{d}\gamma\int F_n\mathrm{d}^2p\,, \quad (20)$$

$$\left[-\mathrm{i}\Omega + \mathrm{i}\theta'(q+n) + p\frac{\partial}{\partial x} - k^2x\frac{\partial}{\partial p}\right]F_n = \sum_{u=1}^{u+v=n}\frac{D_{2u}}{\gamma_0}E_u\frac{\partial F_v}{\partial\gamma}\,. \quad (21)$$

Since we deal with FEL eigenmode problems, the initial value process in Eqs. (20) and (21) can be and have been neglected.

3 Scaling function for linear harmonic gain

Generally, the solution for the n-th harmonic bunching or for the n-th harmonic field amplitude is a combination of two different components, one given by the decoupled linear analysis, and another driven by the exponential instability of the sum of the lower nonlinear harmonic and the fundamental radiation[25]. In Eq. (21), the term $\nu = 0$ represents the linear solution corresponding to the self-amplification of the certain harmonic radiation, and other terms represent the nonlinear harmonic solution corresponding to the strongly bunched electron beam. Simply, one can deal with the two components separately. In this section, we study the linear harmonic generation.

Initially, we assume the electron beam has a uniform longitudinal distribution and a uniform transverse density with the form

$$u(p^2+k^2x^2)=\frac{1}{\pi k^2a^2}\Theta(k^2a^2-p^2-k^2x^2)\,, \qquad (22)$$

where the step function $\Theta(\nu) = 1$ for $\nu > 0$ and $\Theta(\nu) = 0$ for $\nu < 0$, $a = (2k_{\rm s}k_{\rm w})^{1/2}R_0$ is the scaled beam size and R_0 is the beam size. Moreover, a Gaussian distribution around the value of γ_0 with the energy spread σ_γ is applied in energy. Thus, we consider the equilibrium distribution to be the form

$$F_0 = h(\gamma)u(p^2+k^2x^2)\,. \qquad (23)$$

To obtain the linear part of the harmonic radiation, we neglect the nonlinear parts and rewrite Eq. (21):

$$\left[-{\rm i}\Omega+{\rm i}\theta'(q+n)+p\frac{\partial}{\partial x}-k^2x\frac{\partial}{\partial p}\right]F_n = \frac{D_{2n}}{\gamma_0}E_nh'(\gamma)u(p^2+k^2x^2)\,. \qquad (24)$$

Then using Eq. (24) we solve F_n in terms of E_n, and invert this result in Eq. (20) to derive the dispersion relation determining μ_n and $E_n(x)$:

$$\left(\mu_n+\frac{1}{n}\nabla_\perp^2\right)E_n = D_{1n}D_{2n}\int\frac{h'(\gamma)}{\gamma^2}{\rm d}\gamma\int{\rm d}^2pu(p^2+k^2x^2)\times \int{\rm d}s{\rm e}^{-{\rm i}\alpha_ns}E_n(x\cos ks+(p/k)\sin ks)\,. \qquad (25)$$

We define $\alpha_n=\mu_n+(\omega-\omega_r)/\omega_r-n\theta'$. To solve Eq. (25), we must find a good approximation for the transverse dependence of the field and the corresponding eigenvalue μ_n. For FEL process, the critical region is the overlap between the beam and the field, yet the tail discrepancy is less sensitivity. Therefore, a Gaussian approximation will be close to the true field inside the electron beam ($x<a$). As a test function, we choose

$$E_n(x)=\begin{cases}{\rm e}^{-\chi_n\frac{x^2}{2a^2}}, & x\leqslant a,\\ AH_0^{(1)}(x\sqrt{\mu_n}), & x\geqslant a.\end{cases} \qquad (26)$$

$\mathrm{Im}(\mu_n)^{1/2}>0$ is required to satisfy the boundary condition at $x\to\infty$, and $H_0^{(1)}$ is the Hankel function. Then the continuity of the logarithmic derivation at $x=a$ leads to

$$a\sqrt{\mu_n}\frac{H_0'^{(1)}(a\sqrt{\mu_n})}{H_0^{(1)}(a\sqrt{\mu_n})}=-\chi_n\,. \qquad (27)$$

Substituting the trial function into (25), we multiply Eq. (25) by $xE_n(x)$ and integrate from $x=0$ to $x=\infty$[11]. Then we obtain

$$\mu_na^2(1-{\rm e}^{-\chi_n})-\frac{\chi_n}{n}[1-(1-\chi_n){\rm e}^{-\chi_n}] = \int_{-\infty}^0\exp\left[-{\rm i}\left(\frac{\mu_n}{D_n}+\frac{\omega-\omega_r}{\omega_rD_n}\right)s-2\left(\frac{\sigma_\gamma}{D_n}\right)^2s^2\right]\times \left(\frac{1-{\rm e}^{-\eta_+}}{\eta_+}-\frac{1-{\rm e}^{-\eta_-}}{\eta_-}\right)\frac{s{\rm d}s}{\cos(ks/D_n)}\,. \qquad (28)$$

with

$$\eta_\pm = 3{\rm i}s\left(\frac{k}{D_n}\right)(k_{\rm s}\varepsilon)+\frac{\chi_n}{2}\left[1\mp\cos\left(\frac{k}{D_n}s\right)\right],$$
$$D_n = \left(\frac{2Z_0e}{\pi mc^2}\frac{I_p}{\gamma_0}\frac{K^2}{1+K^2/2}\right)^{1/2}[JJ]_n. \qquad (29)$$

Equations (27) and (28) can be numerically solved to determine the complex parameters χ_n and μ_n/D_n. Therefore, the gain function can be expressed in the scaled form

$$\frac{\mathrm{Im}(\mu_n)}{D_n}=\frac{1}{2k_{\rm w}L_{Gn}D_n}=G(k_{\rm s}\varepsilon,\frac{\sigma_\gamma}{D_n},\frac{k_\beta}{k_{\rm w}D_n},\frac{\omega-\omega_{\rm s}}{\omega_{\rm s}D_n})\,. \qquad (30)$$

The scaling parameter D_n is a measure of the transverse electron current. Hence, we present a more universal scaling function for FEL gain, the fundamental and the linear harmonic generation included. When making n=1, our results reduce to the one given by Yu[12].

Starting from the Maxwell-Vlasov equations, we derive a set of self consistent, scaled equations to describe the interaction between the electron beam and the field, the 3D effects and the harmonic radiation included. In comparison with Kim and Huang[27], the relationship between the linear and nonlinear harmonic interactions is more easily read from our equations. Besides, we give a dispersion relation for the linear harmonic gain, which we solved by the variational approximation method[11], while the matrix formulation method[28] is employed by Kim and Huang[27]. What's more, the scaling function presented here provides a useful way to describe the dependence of the FEL gain on the electron parameters. As an example, Fig. 1 shows us a quick estimate of the 3rd linear harmonic gain by covering most of the practical range of the FEL parameters.

The success of our analysis may depend largely on the electron distribution and the choice of the test function. The test function in Eq. (26), actually the fundamental transverse mode E_{00}, is the exact solution outside the electron beam, and inside it has the correct leading behavior[9, 31]. Thus, the intrinsic property of the variational approximation method[11] assures that the error of the eigenvalue μ_n depends quadratically on the small errors in the test function. More generally, our analysis can be extended to any kinds of initial electron distribution[14, 15] and other high order transverse mode[31].

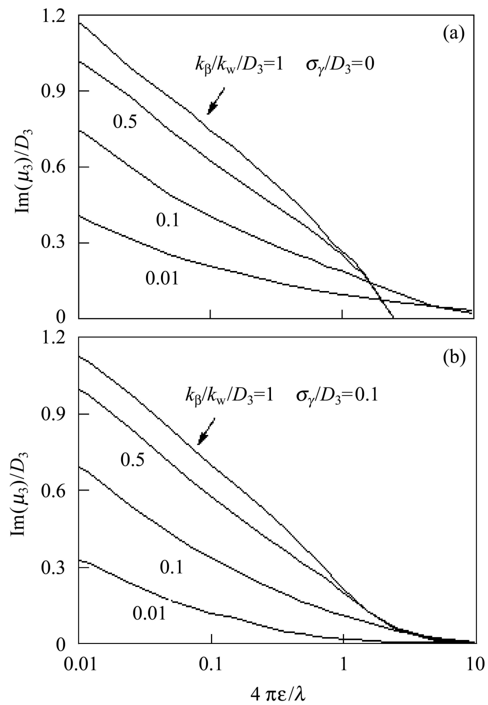

Fig. 1. Scaling function for the 3rd linear harmonic gain vs. scaled emittance for several values of $k_\beta/k_w/D_3$, corresponding to scaled energy spread (a) $\sigma_\gamma/D_3=0$ and (b) $\sigma_\gamma/D_3=0.1$ with the optimal detuning.

4 The nonlinear harmonic generation

The nonlinear harmonic generation occurs when electron beam is strongly bunched in the ponderomotive potential formed by the undulator field and the radiation field of the fundamental frequency. In order to determine the nonlinear harmonic interactions, we consider only the nonlinear components and neglect the linear component in Eq. (21). Thus, we rewrite Eq. (21)

$$\left[-\mathrm{i}\Omega+\mathrm{i}\theta'(q+n)+p\frac{\partial}{\partial x}-k^2x\frac{\partial}{\partial p}\right]F_n=\sum_{v=1}^{u+v=n}\frac{D_{2u}}{\gamma_0}E_u\frac{\partial F_v}{\partial\gamma}\,. \tag{31}$$

Now we could deal with each order of the nonlinear harmonic generation in detail. Some further assumptions such as that the fateful contribution to the nonlinear harmonic generation is from the fundamental radiation and the even harmonic on the right hand of Eq. (31) is neglected are applied before we go on. Then for the 3rd nonlinear harmonic generation one may obtain a dispersion relation

$$E_3\propto D_{13}D_{21}^3E_1^3\,. \tag{32}$$

One could also observe that, for the 5th harmonic, the leading nonlinear terms are

$$E_5\propto D_{15}(D_{21}^5E_1^5+D_{21}^2E_1^2D_{23}E_3)\,. \tag{33}$$

From these relationships, we can conclude that the nonlinear harmonic generation grows faster than the fundamental radiation, and the gain length scales inversely with the harmonic number. Therefore, people are interested in the nonlinear harmonic generation, which can have significant power and offer shorter wavelength than the fundamental, thus extending the applications of X-ray FEL facilities[27].

5 Numerical results

In this section, we shall study the harmonic contents of two FEL examples, in DUV and X-ray spectral regions respectively. The nominal parameters are listed in Table 1, and the steady state simulations up to the 3rd harmonic are performed by 3D FEL code TDA-H[22] which is upgraded from TDA3D[3]. Figs. 2 and 3 show us the power growth of the fundamental and the 3rd harmonic component as a function of the radiator length for the nominal parameters of DUV FEL and X-ray FEL example, where the region of the linear harmonic and the nonlinear harmonic interaction can be well recognized. Hence, we can obtain the gain length of the fundamental and the harmonic radiation easily.

Table 1. Nominal parameters of DUV and X-ray FEL.

	DUV	X-ray
electron beam and undulator		
λ_{s1}/nm	262	3
λ_{s3}/nm	87.3	1
E/MeV	160	1200
I_p/A	300	2000
ε/(μm·rad)	6	2
$\sigma_\gamma/\gamma(\times10^{-4})$	1	2
β/m	1.5	8
λ_w/cm	2.5	2
K	1.41	1.14
calculated FEL performances		
ρ	0.0026	0.0010
L_{G1}/m	0.53	1.53
L_{G3}/m	1.24	6.65

We compare the simulation results with the analytical estimates introduced in this paper. Firstly, we obtain that due to lower coupling coefficient the 3rd linear harmonic radiation grows much more slowly than the fundamental and performs more sensitive

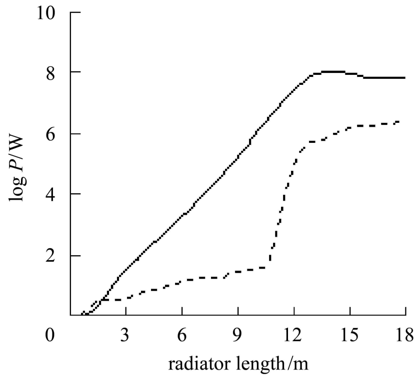

Fig. 2. The fundamental (262 nm, the solid) and the 3rd harmonic radiation (87.3 nm, the dashed) power growth for the nominal parameters of DUV FEL example.

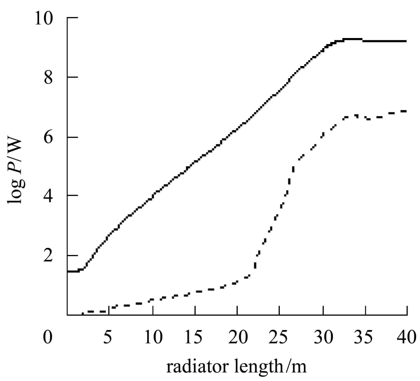

Fig. 3. The fundamental (3 nm, the solid) and the 3rd harmonic radiation (1 nm, the dashed) power growth for the nominal parameters of X-ray FEL example.

to the energy spread, peak current and emittance of electron beam than that of the fundamental radiation. Moreover, we observe that the gain length of the 3rd nonlinear harmonic radiation is basically one third of the fundamental, actually a little bit larger. We also find that the 3rd nonlinear harmonic interaction is very sensitive to the "warm-beam" effects. With the degradation of beam quality, one even can't clearly differentiate the linear and nonlinear harmonic region. When we validate the analytical approach with 3D simulation results for both DUV and X-ray FEL example, the agreement for most of the practical range of FEL parameters is good to within a factor of 0.95.

6 Concluding remarks

In this paper, we describe an analytical approach to solve the interaction between the electron beams and the radiation fields which determines the gain length of the fundamental and the odd harmonic generation in high gain FEL, including the "warm-beam" effects and the optical effects of radiation field. Our object is to provide a simple physical picture of the gain of each harmonic radiation. We manage to develop a universal scaling function to evaluate the dependence of the gain on the physical parameters, and it is checked by 3D FEL simulation. Our analytical approach is consistent with the simulation results in large range. It will be very useful in the optimization of the FEL design.

We focus on the odd harmonic radiation in this paper, thus, the even harmonic generation is neglected in this discussion. Therefore, for obtaining the realistic results, further study should be undertaken.

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