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# Exotic Structures of Odd-*A* Carbon Isotopes in the Deformed Relativistic Mean-Field Theory*

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***Abstract*** *We study contributions of the pion meson and spatial component of the omega meson in the odd-A carbon isotopes. The pion and spatial omega provide small attractions in odd-A nuclei, giving rise to considerable influences on the single-particle energies rather than the bulk properties such as total binding energies, and root-mean-square (rms) radii. The* $\pm\Omega$ *(spin) splittings, arising from the spatial omega, are large in* $^{11}C$ *and* $^{13}C$ *and drop as the isospin rises in odd-A carbon isotopes. As an isovector, the pion can shift slightly the relative potential depth of neutron and proton, contrary to the role of the rho meson. There is a general trend that both the pion and spatial omega fields reduce with the rise of isospin in the isotopic chain. From the normal nucleus to halo nucleus, an abnormal drop of the pion or spatial omega field may occur, as can be seen in* $^{19}C$*,* $^{15}C$*, and* $^{21}C$*.*

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## 1 Introduction

Nuclei and nuclear matter are quantum many-body systems where one nucleon interacts with others through the exchange of virtual mesons. In the past years, many theoretical approaches were developed to describe the nuclear many-body system and depict the characteristics of nuclei extensively to different degrees. In particular, the relativistic quantum field theory, which was well developed during 1970s', has been applied to the nuclear many-body problem (for a review, see Ref. [1]). Not only does the theory has a merit of relativistic covariance, it can also reproduce the spin-related physics such as the spin-orbit splittings and pseudo-spin symmetry automatically. The Walecka model,[2] which was raised in early 1970's, is a relativistic quantum field model at the first stage, including nucleons, scalar and vector mesons. The main feature of the model is that there exist strong scalar mesons, and vector meson fields at usual nuclear matter density, where the former provides intermediate-range attraction and the latter short-range repulsion. Though the model can describe self-consistently properties of the bulk and single-particle of nuclei to a certain degree, it suffers from two main drawbacks, one of which is that it has quite a large incompressibility and another is that it is not able to describe the binding and root-mean-square (rms) radius of nuclei well simultaneously (for instance, see Ref. [3]). Moreover, the quantum field theory on the hadron level itself encounters basic difficulties in considering the renormalization and the role of interior degrees of freedom in nucleons. Therefore, different effective relativistic mean-field (RMF) models have been raised to describe the nuclear phenomenology.[4−13] There are two important properties of the effective RMF models: the self-consistency of the calculation and the inclusion of non-linear terms of meson fields.

The parameters of the effective RMF models are obtained from fitting the saturation properties of nuclear matter and ground-state properties of some spherical nuclei. The effective RMF models can therefore reproduce well many ground-state and low-excited-state properties of spherical nuclei. As a further step, the RMF theory has been extended to study the deformed nuclei,[14−18] where only the projection of total angular momentum is the good quantum number. In Refs. [14] and [15], the deformed RMF (DRMF) model is solved through the basis expansion of the harmonious oscillator. In Ref. [16], the angular momentum projection method is applied to obtain the good quantum number of the deformed nuclei. The deformed RMF Dirac wave functions in Refs. [17] and [18] are resolved through the expansion of a nearby spherical nuclear basis with the potentials given in the coordinate space. Nuclear magnetic moments of some light odd-$A$ nuclei are studied via inclusion of the spatial part of vector meson in Refs. [18] $\sim$ [20], and, especially in Ref. [18], the pion meson is taken into account. For the spin saturated nuclei, the contribution from the negative-parity mesons is nothing in DRMF, while for the deformed odd-$A$ nuclei, the spin of the valence nucleon has not paired so that the time reversal symmetry is broken, leading to non-zero contributions of negative-parity mesons. For the odd-$A$ nuclei

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we will take into account the pion and spatial omega together in DRMF. Unlike the treatment in Refs. [17] and [18], we will solve the equations of motion of nucleons and mesons both in an expansion method with the deformed harmonic-oscillator basis.[15] It should be pointed out that to investigate the contribution of negative-parity mesons one should include the Fork terms in the relativistic framework (e.g. see Refs. [1] and [21]). However, this is beyond the RMF framework, where the Fock term is not involved.

Besides the nuclei of $\beta$-stability, some nuclei near drip lines have been recently studied via the DRMF models.[22–24] It is significant to take into account the pion meson in nuclei of this region based on the considerations that the pion provides attraction, which would bring about influences on the diffusive density distributions of exotic nuclei due to its long interaction range and isospin-dependent effects. It would be natural to introduce the pion as a Goldstone boson of chiral symmetry. However, the constraint from chiral symmetry may cause additional complications in the effective models in describing nuclear bulk properties. Therefore, the pion exchange is directly introduced as an effective interaction. In the effective Lagrangian, the pseudo-vector (PV) coupling of pion meson is preferred, since the pseudo-scalar coupling gives rise to the abnormally large self-energies of nucleons and $s$-wave scattering length of pion-nucleon scattering (see Ref. [1] and references therein).

In this paper we would like to learn what role the pion will play for the formation of halos or skins. Amongst so many isotopic chains, we will choose carbon isotopes for example. In carbon isotopes, there are abundant halo and skin phenomena (for recent reviews, please see Refs. [22] and [25] and references therein), which can be taken properly as theoretical laboratories to investigate the role of the pion. The arrangement of the paper is as follows. In Sec. 2, we give the RMF formalism briefly. In Sec. 3, the numerical results and the discussion are presented. Summaries are given in Sec. 4. The formulations for the pion and spatial component of vector mesons in DRMF are presented in the appendix.

## 2 Formalism

We will give a brief RMF description for finite nuclei in this section. The formalism includes the Lagrangian, equations of motion for nucleons and mesons. The effective Lagrangian as the starting footing of nuclear many-body system is

$$\begin{aligned}\mathcal{L} = \bar{\psi}\Big[&\mathrm{i}\gamma_\mu\partial^\mu - M_N + g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_3 b_0^\mu \\ &- \frac{f_\pi}{m_\pi}\gamma^5\gamma^\mu\tau\cdot\partial_\mu\pi - e\frac{1}{2}(1+\tau_3)\gamma_\mu A^\mu\Big]\psi + U(\sigma) \\ &+ \frac{1}{2}(\partial_\mu\pi\partial^\mu\pi - m_\pi^2\pi^2) + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ \frac{1}{2}m_\rho^2 b_{0\mu}b_0^\mu - \frac{1}{4}A_{\mu\nu}A^{\mu\nu}\,, \end{aligned} \tag{1}$$

where $\psi$, $\sigma$, $\omega$, $b_0$, and $\pi$ are the fields of the nucleon, scalar, vector, neutral isovector-vector, and pseudoscalar mesons, with their masses $M_N$, $m_\sigma$, $m_\omega$, $m_\rho$, and $m_\pi$ respectively. $A_\mu$ and $A_{\mu\nu}$ are respectively the photon field and its field strength tensor, $g_i$ $(i=\sigma,\omega,\rho)$ and $f_\pi$ are the corresponding meson-nucleon couplings, $\tau$ and $\tau_3$ is the isospin Pauli matrix and its third component, and $F_{\mu\nu}$, $B_{\mu\nu}$, and $A_{\mu\nu}$ are the strength tensors of $\omega$ and $\rho$ mesons and photon, respectively,

$$\begin{aligned}&F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu\,, \qquad B_{\mu\nu} = \partial_\mu b_{0\nu} - \partial_\nu b_{0\mu}\,, \\ &A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \end{aligned} \tag{2}$$

The self-interacting potential of sigma meson reads

$$U(\sigma) = \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4\,, \tag{3}$$

In RMF, only $b_0$ and $\pi_0$, the third isospin components of isovector mesons $\rho$ and $\pi$, may have non-zero contributions due to the charge conservation. We use the PV coupling for the pion meson. Though it is not renormalizable, we remind ourselves here that we work on the effective interactions. The pseudo-vector coupling $f_\pi$ is taken as its usually used value 0.9708.

Using the Euler–Lagrangian equation, the Dirac equation of motion is given as

$$\begin{aligned}\Big(&\mathrm{i}\gamma_\mu\partial^\mu - M_N + g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_3 b_0^\mu \\ &-\frac{f_\pi}{m_\pi}\gamma^5\gamma^\mu\boldsymbol{\tau_3}\cdot\partial_\mu\pi_0 - e\frac{1}{2}(1+\tau_3)\gamma_\mu A^\mu\Big)\psi_i = 0\,. \end{aligned} \tag{4}$$

The mesons and photon obey the following equations,

$$(\partial^2 + m_\sigma^2)\sigma = g_\sigma\sum_i^A\bar{\psi}_i\psi_i - g_2\sigma^2 - g_3\sigma^3\,, \tag{5}$$

$$\partial^\mu F_{\mu\nu} + m_\omega^2\omega_\nu = g_\omega\sum_i^A\bar{\psi}_i\gamma_\nu\psi_i\,, \tag{6}$$

$$\partial^\mu B_{\mu\nu} + m_\rho^2\rho_\nu = g_\rho\sum_i^A\bar{\psi}_i\tau_3\gamma_\nu\psi_i\,, \tag{7}$$

$$(\partial^2 + m_\pi^2)\pi_0 = \frac{f_\pi}{m_\pi}\sum_i^A\partial_\mu(\bar{\psi}_i\gamma^5\gamma^\mu\tau_3\psi_i)\,, \tag{8}$$

$$\partial^\mu A_{\mu\nu} = e\sum_i^A\bar{\psi}_i\frac{1+\tau_3}{2}\gamma_\nu\psi_i\,, \tag{9}$$

where $A$ is the nucleon number of the system.

In the RMF approximation, the Dirac equation can be written explicitly as

$$\begin{aligned}(&-\mathrm{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta M_N^* + g_\omega\omega^0(\boldsymbol{r}) - g_\omega\boldsymbol{\alpha}\cdot\boldsymbol{\omega}_v(\boldsymbol{r}) + g_\rho\tau_3 b_0^0(\boldsymbol{r}) \\ &- g_\rho\tau_3\boldsymbol{\alpha}\cdot\boldsymbol{b}_0(\boldsymbol{r}) + \frac{f_\pi}{m_\pi}\gamma^5\tau_3\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}\pi_0(\boldsymbol{r}) \\ &+ e\frac{1}{2}(1+\tau_3)A^0(\boldsymbol{r}))\psi_i(\boldsymbol{r}) = 0 \end{aligned} \tag{10}$$

with $M_N^* = M_N - g_\sigma\sigma(\boldsymbol{r})$. Here the spatial vector of the electro-magnetic field is neglected since it is much smaller than the counterpart of vector meson in the strong interaction system. For mesons and photon, equations become

$$(-\Delta + m_\sigma^2)\sigma(\boldsymbol{r}) = g_\sigma \sum_i^A \bar{\psi}_i(\boldsymbol{r})\psi_i(\boldsymbol{r}) - g_2\sigma^2(\boldsymbol{r}) - g_3\sigma^3(\boldsymbol{r}) = g_\sigma\rho_s(\boldsymbol{r}) - g_2\sigma^2(\boldsymbol{r}) - g_3\sigma^3(\boldsymbol{r}) \,, \tag{11}$$

$$(-\Delta + m_\omega^2)\omega^0(\boldsymbol{r}) = g_\omega \sum_i^A \bar{\psi}_i(\boldsymbol{r})\gamma^0\psi_i(\boldsymbol{r}) = g_\omega\rho_B(\boldsymbol{r}) \,, \tag{12}$$

$$(-\Delta + m_\rho^2)b_0^0(\boldsymbol{r}) = g_\rho \sum_i^A \bar{\psi}_i(\boldsymbol{r})\tau_3\gamma^0\psi_i(\boldsymbol{r}) = g_\rho\rho_3(\boldsymbol{r}) \,, \tag{13}$$

$$(-\Delta + m_\pi^2)\pi_0(\boldsymbol{r}) = \frac{f_\pi}{m_\pi} \sum_i^A \boldsymbol{\nabla}\cdot(\bar{\psi}_i(\boldsymbol{r})\gamma^5\boldsymbol{\gamma}\tau_3\psi_i(\boldsymbol{r})) \,, \tag{14}$$

$$-\Delta A^0 = e\sum_i^A \bar{\psi}_i(\boldsymbol{r})\frac{1+\tau_3}{2}\gamma^0\psi_i(\boldsymbol{r}) = e\rho_p(\boldsymbol{r}) \,. \tag{15}$$

For the spatial parts of vectors, they have only the angular part

$$(-\Delta + m_\omega^2)\boldsymbol{\omega}_\varphi(\boldsymbol{r}) = g_\omega \sum_i^A \bar{\psi}_i(\boldsymbol{r})\boldsymbol{\gamma}\psi_i(\boldsymbol{r}) = g_\omega \boldsymbol{j}_{\omega_\varphi}(\boldsymbol{r}) \,, \tag{16}$$

$$(-\Delta + m_\rho^2)\boldsymbol{b}_{0\varphi}(\boldsymbol{r}) = g_\rho \sum_i^A \bar{\psi}_i(\boldsymbol{r})\tau_3\boldsymbol{\gamma}\psi_i(\boldsymbol{r}) = g_\rho \boldsymbol{j}_{b_\varphi}(\boldsymbol{r}) \,, \tag{17}$$

where $\rho_s$, $\rho_B$, $\rho_3$, and $\rho_p$ are different densities, and $\boldsymbol{j}$'s are vector currents. In the spherical or spin-saturated deformed nuclei, the spatial parts of vector mesons vanish. In the appendix, we will give contributions of the pion and spatial vector mesons in the axially symmetric frame for odd-$A$ nuclei and show how their contributions vanish for even-even nuclei.

The total binding energy is written as

$$\begin{aligned} E_{\text{total}} &= E_N + E_\sigma + E_{\sigma NL} + E_{\omega_0} + E_{b_0} + E_c + E_{\boldsymbol{\omega}_v} + E_{\boldsymbol{b}_v} + E_{\text{CM}} + E_\pi \\ &= \sum_\alpha (E_\alpha - M_N) - \frac{1}{2}\int \mathrm{d}^3\boldsymbol{r}\Big(g_\sigma\sigma(\boldsymbol{r}) + \frac{2}{3}g_2\sigma^3(\boldsymbol{r}) + \frac{1}{2}g_3\sigma^4(\boldsymbol{r})\Big) - \frac{1}{2}g_\omega\int \mathrm{d}^3\boldsymbol{r}\omega_0(\boldsymbol{r})\rho_B(\boldsymbol{r}) \\ &\quad - \frac{1}{2}g_\rho\int \mathrm{d}^3\boldsymbol{r} b_0(\boldsymbol{r})\rho_3(\boldsymbol{r}) + \frac{1}{2}g_\omega\int \mathrm{d}^3\boldsymbol{r}\boldsymbol{\omega}_\varphi(\boldsymbol{r})\cdot\boldsymbol{j}_{\omega_\varphi}(\boldsymbol{r}) + \frac{1}{2}g_\rho\int \mathrm{d}^3\boldsymbol{r}\boldsymbol{b}_\varphi(\boldsymbol{r})\cdot\boldsymbol{j}_{b\varphi}(\boldsymbol{r}) \\ &\quad - \frac{1}{2}e\int \mathrm{d}^3\boldsymbol{r}A_0(r)\rho_c(r) - \frac{3}{4}41A^{1/3} + E_\pi \,, \end{aligned} \tag{18}$$

where $E_\pi$ will be given in the appendix. The pairing correlation is not included above, since the pairing correlation is small due to the Pauli blocking in odd-$A$ nuclei.

## 3 Results of Odd-$A$ Carbon Isotopes

For the spherical system, the meson fields can be easily obtained through integrations of Green functions. In the deformed system, we refer to the treatments of Ref. [15], namely, solve the Dirac equations together with meson equations by the expansion in the harmonic-oscillator basis. The Dirac and meson equations are expanded separately in the harmonic-oscillator basis with the respective oscillator quantum number $N_F$ and $N_B$, and we take $N_F = N_B = 12$.

We use two parameter sets of RMF model NL-SH[11] and NL3[13] to perform calculations. The parameter set NL-SH is as follows. $M_N = 939.0$ MeV, $m_\sigma = 526.059$ MeV, $m_\omega = 783.0$ MeV, $m_\rho = 763.0$ MeV, $g_\sigma = 10.4436$, $g_\omega = 12.9451$, $g_\rho = 4.3828$, $g_2 = -6.9099$ fm$^{-1}$, and $g_3 = -15.8337$. The parameter set NL3 is $M_N = 939.0$ MeV, $m_\sigma = 508.194$ MeV, $m_\omega = 782.501$ MeV, $m_\rho = 763.0$ MeV, $g_\sigma = 10.217$, $g_\omega = 12.868$, $g_\rho = 4.474$, $g_2 = -10.431$ fm$^{-1}$, and $g_3 = -28.885$. It is significant to show the model sensitivity of the results by comparing results obtained from the NL3 and NL-SH parameter sets.

To testify the modifications due to the pion inclusion in the DRMF code, we perform calculations at first for nuclei $^{17}$O with parameter set NL-SH. The calculated field and particle energies, deformation parameter $\beta$, and root-mean-square (rms) radii are given in Table 1. The label Def. 0 stands for the deformed solution with the time reversal symmetry, Def. 1 stands for the case where the time reversal symmetry is broken by the vector current, and Def. 2 is similar to Def. 1 but with the pion meson included. To focus on more important factors, we ignore the spatial $\rho$ meson in the calculation since its contribution is much smaller. Both of the pion and spatial omega provide small attraction, and that can be seen in Table 1: the sign of the pion and spatial omega field energies is the same as that of the field energy of the $\sigma$-meson, which provides the attraction. The pion source term is mainly from the single valence nucleon, and hence the pion produces just a small long-range attraction. Deformations with both the Def. 1 and Def. 2 of $^{17}$O are small under the configurations given in Table 2. As seen in Table 2, the pion may change the single-particle binding energy by 0.5 MeV or so. Since the pion is an isovector meson, it shifts the neutron and proton potentials inversely. The non-zero vector current in odd-$A$ nuclei breaks the $\pm\Omega$ degeneracy, shown in Table 2. The similar splittings of $^{17}$O with the NL1 parameter set were given in Ref. [20], where the spatial omega was considered. As seen in Table 2 the pion has almost negligible effects in $\pm\Omega$ splittings.

The valence neutron in odd-$A$ carbon isotopes is more and more weakly-bound as the isotope goes to the neutron drip line. As expected, the pion contribution in these nuclei is supposed to be important, since the pion has a long range of interaction. Also, it is interesting to investigate its isospin-dependent effects because the pion is a isovector. Table 3 gives the calculated field and particle energies, deformation parameter $\beta$, and rms radii for odd-$A$ nuclei with NL-SH set, which are consistent with the calculations in Ref. [22]. Similar in $^{17}$O, the spatial vector current and the pion contribute small attractions to the total binding of the system as given in Table 3. Small rises of the binding energy and small shrinks of the rms radii are found in these odd-$A$ nuclei as the pion and spatial omega are included. The deformation parameter $\beta$ is almost unchanged by the vector current, since the cancellation exists for the particles occupying the $\pm\Omega$ states. The isovector pion, which gives rise to respectively the coherent shifts of the mean fields of neutron and proton, causes a small shift in nuclear deformation, shown in the table. However, the pion and spatial omega undergo generally the decreasing trend with the rise of isospin, which is beyond expectation. Especially, the pion or spatial omega has an abnormal drop in some nuclei. We will substantiate this in the following.

**Table 1** The calculated field and particle energies (in MeV), deformation parameter $\beta$, and matter rms radii (in fm) with NL-SH set for $^{17}$O. The definitions of label Def. 0, Def. 1 and Def. 2 are given in the context. The experimental binding energy of $^{17}$O is 131.76 MeV.

| Nucleus | | $E_N$ | $E_\sigma$ | $E_\omega$ | $E_{\omega_\varphi}$ | $E_\pi$ | $E_{\rm total}$ | $\beta$ | $r_{\rm rms}$ |
|---|---|---|---|---|---|---|---|---|---|
| | Def. 0 | −400.69 | 2057.51 | −1723.57 | | | −133.15 | 0.037 | 2.620 |
| $^{17}$O | Def. 1 | −401.60 | 2061.18 | −1726.83 | 0.15 | | −133.28 | 0.018 | 2.616 |
| | Def. 2 | −402.23 | 2062.90 | −1728.40 | 0.16 | 0.28 | −134.00 | 0.022 | 2.615 |

**Table 2** The calculated single-particle energies (in MeV) for $^{17}$O with Def. 1 and Def. 2. The parameter set is NL-SH.

| Nucleus | | $(1/2)^+_1$ | $(1/2)^-_1$ | $(3/2)^-_1$ | $(1/2)^-_2$ | $(3/2)^+_1$ | $(1/2)^+_2$ |
|---|---|---|---|---|---|---|---|
| $^{17}$O | Def. 1, $n$ | −42.31 | −22.48 | −22.49 | −15.92 | | |
| | | −42.57 | −23.02 | −22.78 | −16.05 | −5.51 | |
| | Def. 1, $p$ | −39.61 | −20.28 | −20.13 | −13.62 | | |
| | | −39.86 | −20.81 | −20.43 | −13.74 | | |
| | Def. 2, $n$ | −42.72 | −22.96 | −22.83 | −15.58 | | |
| | | −42.97 | −23.50 | −23.13 | −15.69 | −5.93 | |
| | Def. 2, $p$ | −39.23 | −19.88 | −19.77 | −13.98 | | |
| | | −39.48 | −20.41 | −20.07 | −14.11 | | |

**Table 3** The same as Table 1 but for carbon isotopes. Here $\beta_n$ and $\beta_p$ are the deformation parameters of neutron and proton, respectively.

| Nucleus | | $E_N$ | $E_{\omega_\varphi}$ | $E_\pi$ | $E_{\rm total}$ | $\beta_n$ | $\beta_p$ | $r_{\rm rms}$ |
|---|---|---|---|---|---|---|---|---|
| $^{11}$C | Def. 0 | −251.20 | | | −73.48 | 0.326 | 0.247 | 2.342 |
| | Def. 1 | −252.57 | 0.36 | | −73.84 | 0.325 | 0.244 | 2.337 |
| | Def. 2 | −252.30 | 0.35 | 0.43 | −74.13 | 0.304 | 0.272 | 2.341 |
| $^{13}$C | Def. 0 | −312.10 | | | −97.38 | 0.002 | 0.002 | 2.402 |
| | Def. 1 | −313.14 | 0.42 | | −97.72 | 0.013 | 0.014 | 2.398 |
| | Def. 2 | −314.45 | 0.55 | 0.58 | −99.80 | 0.022 | −0.021 | 2.394 |
| $^{15}C$ | Def. 0 | −344.56 | | | −107.67 | 0.294 | 0.156 | 2.596 |
| | Def. 1 | −345.08 | 0.11 | | −107.74 | 0.284 | 0.157 | 2.591 |
| | Def. 2 | −346.05 | 0.12 | 0.33 | −108.25 | 0.290 | 0.170 | 2.588 |
| $^{17}$C | Def. 0 | −380.19 | | | −113.07 | 0.529 | 0.284 | 2.770 |
| | Def. 1 | −380.72 | 0.13 | | −113.22 | 0.527 | 0.284 | 2.768 |
| | Def. 2 | −381.45 | 0.13 | 0.22 | −113.33 | 0.518 | 0.279 | 2.762 |
| $^{19}$C | Def. 0 | −406.02 | | | −116.31 | 0.424 | 0.264 | 2.944 |
| | Def. 1 | −406.22 | 0.05 | | −116.36 | 0.424 | 0.264 | 2.943 |
| | Def. 2 | −406.46 | 0.06 | 0.07 | −116.55 | 0.422 | 0.259 | 2.942 |
| $^{21}$C | Def. 0 | −437.96 | | | −119.08 | 0.126 | 0.104 | 3.014 |
| | Def. 1 | −438.29 | 0.09 | | −119.16 | 0.116 | 0.101 | 3.012 |
| | Def. 2 | −438.60 | 0.09 | 0.07 | −119.56 | 0.116 | 0.087 | 3.012 |

Note that the source terms of the pion and spatial omega are mainly from the single valence nucleon. As can be seen in Eq. (A15) in the appendix, the source term of the spatial omega is calculated from the product of the spin "up" and "down" components of the Dirac spinor. Therefore, the spatial vector meson vanishes for spherically odd-$A$ nuclei. As seen in Eq. (A20) in the appendix, the source term of pion consists of two parts, the

$z$-component and the perpendicular parts. The former does not vanish even for spherically odd-$A$ nuclei, as seen in Eq. (A22). The latter is from the product of the spin ‘up’ and ‘down’ components of spinor. For the unpaired nucleons with diffusive spatial extension, the source term of the pion and spatial omega will almost vanish for the derivatives in the source terms, or, due to the fact that the valence nucleon has a dominant spin ‘‘up” or “down” component. Therefore, the general trend is that contributions of the pion and spatial omega decrease with the rise of isospin in the isotopes, as the last single nucleon becomes less and less bound. The pion meson plays very limited role although the pion may provide the long-range attraction. As seen in Table 3, the energy of the spatial omega $E_{\omega_\varphi}$ has the abnormal decrease in $^{15}$C and $^{19}$C, while the decrease is much obvious in $^{19}$C. The abnormal decrease of the pion energy $E_\pi$ also occurs in $^{19}$C. For the odd-$A$ halo nuclei, the attraction from the pion is not enhanced but reduced, and it is thus helpful for the halo formation. The abnormal decrease of the pion requires the occurrence of the very weak binding and abnormally diffusive distribution of the valence nucleon. Thus, the abnormal decrease of the pion or the spatial omega may serve as the signature of the formation of halo in nuclei far off $\beta$-stability.

**Table 4** The same as Table 3 but with NL3.

| Nucleus | | $E_N$ | $E_{\omega_\varphi}$ | $E_\pi$ | $E_{\text{total}}$ | $\beta_n$ | $\beta_p$ | $r_{\text{rms}}$ |
|---|---|---|---|---|---|---|---|---|
| $^{11}$C | Def. 0 | −259.62 | | | −73.91 | 0.283 | 0.209 | 2.323 |
| | Def. 1 | −262.60 | 0.37 | | −74.27 | 0.279 | 0.203 | 2.312 |
| | Def. 2 | −260.08 | 0.35 | 0.45 | −74.57 | 0.265 | 0.241 | 2.325 |
| $^{13}$C | Def. 0 | −321.34 | | | −98.06 | 0.001 | 0.000 | 2.393 |
| | Def. 1 | −322.02 | 0.38 | | −98.36 | 0.010 | 0.008 | 2.389 |
| | Def. 2 | −321.10 | 0.50 | 0.53 | −100.23 | 0.020 | −0.020 | 2.389 |
| $^{15}$C | Def. 0 | −342.40 | | | −108.38 | 0.304 | 0.1561 | 2.620 |
| | Def. 1 | −342.82 | 0.09 | | −108.46 | 0.302 | 0.152 | 2.617 |
| | Def. 2 | −343.48 | 0.10 | 0.31 | −108.85 | 0.303 | 0.167 | 2.611 |
| $^{17}$C | Def. 0 | −375.64 | | | −113.96 | 0.545 | 0.278 | 2.801 |
| | Def. 1 | −376.10 | 0.11 | | −114.07 | 0.545 | 0.278 | 2.799 |
| | Def. 2 | −377.07 | 0.11 | 0.20 | −114.17 | 0.529 | 0.272 | 2.791 |
| $^{19}$C | Def. 0 | −402.71 | | | −117.74 | 0.425 | 0.254 | 2.966 |
| | Def. 1 | −402.98 | 0.07 | | −117.80 | 0.425 | 0.253 | 2.964 |
| | Def. 2 | −403.18 | 0.07 | 0.08 | −117.93 | 0.419 | 0.248 | 2.963 |
| $^{21}$C | Def. 0 | −431.09 | | | −121.26 | 0.159 | 0.134 | 3.076 |
| Def. 1 | −431.58 | 0.12 | | −121.38 | 0.159 | 0.133 | 3.074 | |
| | Def. 2 | −431.77 | 0.12 | 0.14 | −121.82 | 0.178 | 0.131 | 3.076 |

Table 4 gives the results with the parameter set NL3. Analogous conclusions to those with the NL-SH set can be drawn by the NL3 set. As seen in Table 4, the pion and spatial omega provide the attraction, the pion can slightly shift the nuclear deformation, and the generally decreasing trend and abnormal decrease of the pion and spatial omega field energies are also found with the NL3, similar to those with the NL-SH.

In odd-$A$ nuclei, the non-zero vector current of nucleon breaks the $\pm\Omega$ degeneracy. As seen in Table 5, the $\pm\Omega$ splitting in carbon isotopes has three explicit features. First, the $\pm\Omega$ splittings in $^{11}$C and $^{13}$C due to the nonzero vector current are quite large with the value about 1.7 MeV. The reason may be attributed to the existence of the hole in the closed shell and sub-shell, considering that the holes in the closed shell and sub-shell at $N = 6$ and $N = 8$ give rise to strong polarization of the core. Second, for nuclei close to the drip line, the $\pm\Omega$ splittings near the Fermi surface are much smaller than those deep in potential well. The polarization caused by the vector current in nuclei near the drip line mainly exists in the deeply internal core. This is due to the fact that the omega meson has quite a short range of interaction. Third, the shift of $\pm\Omega$ splitting by the pion inclusion is rather small.

As seen in Table 5, the pion may have its visible role on the valence nucleon binding energy due to its long-range interaction and relative shift of the potential depths of neutron and proton due to its isovector feature. The $\rho$ meson, which is an isovector, deepens the potential well of proton and shallows that of neutron. The pion, being isovector scalar meson, plays the contrary role to that of rho meson, say, it shifts down the potential depth of neutron and shifts up that of proton. The potential shift has an obvious effect on the binding energy of the valence nucleon. We may see from the binding energy shown in Table 5 that the shift of the potential depth due to the pion inclusion is small. Near the closed shell, the shift is much larger than other regions. For nuclei $^{19}$C and $^{21}$C, which are close to the drip line, the shift is almost negligible. Generally, the pion can cause the rise of the binding energy of the valence neutron. But its role on valence neutrons of nuclei close to the drip line is almost negligible. We may find also that the pion plays a role of little importance in neutron halo extension of nuclei near the drip line and it can only shrink the diffusive valence neutron distribution a little bit.

**Table 5** The calculated single-particle energies (in MeV) for odd-$A$ carbon isotopes for Def. 1 and Def. 2 with the NL-SH parameter set.

| Nucleus | | $(1/2)^+_1$ | $(1/2)^-_1$ | $(3/2)^-_1$ | $(1/2)^-_2$ | $(1/2)^+_2$ | $(3/2)^+_1$ | $(1/2)^+_3$ | $(5/2)^+_1$ |
|---|---|---|---|---|---|---|---|---|---|
| $^{11}$C | Def. 1, $n$ | −42.34 | −18.84 | −16.11 | | | | | |
| | | −40.97 | −18.58 | | | | | | |
| | Def. 1, $p$ | −35.79 | −13.51 | −10.01 | | | | | |
| | | −34.43 | −13.27 | −8.72 | | | | | |
| | Def. 2, $n$ | −42.43 | −18.50 | −16.27 | | | | | |
| | | −41.02 | −18.23 | | | | | | |
| | Def. 2, $p$ | −35.56 | −13.91 | −9.83 | | | | | |
| | | −34.22 | −13.70 | −8.54 | | | | | |
| $^{13}$C | Def. 1, $n$ | −41.05 | −17.61 | −17.15 | | | | | |
| | | −42.71 | −17.94 | −18.06 | −9.5 | | | | |
| | Def. 1, $p$ | −40.13 | −16.72 | −16.25 | | | | | |
| | | −41.77 | −17.06 | −17.18 | | | | | |
| | Def. 2, $n$ | −41.06 | −16.45 | −17.50 | | | | | |
| | | −42.95 | −16.73 | −18.48 | −11.14 | | | | |
| | Def. 2, $p$ | −39.59 | −17.93 | −16.05 | | | | | |
| | | −41.28 | −18.33 | −16.97 | | | | | |
| $^{15}$C | Def. 1, $n$ | −40.09 | −19.46 | −19.46 | −10.07 | −2.76 | | | |
| | | −39.90 | −19.35 | −16.91 | −9.91 | | | | |
| | Def. 1, $p$ | −43.08 | −22.21 | −19.75 | | | | | |
| | | −42.88 | −22.08 | −19.61 | | | | | |
| | Def. 2, $n$ | −39.56 | −19.85 | −16.50 | −9.90 | | | | |
| | | −39.76 | −19.99 | −16.63 | −10.07 | −3.55 | | | |
| | Def. 2, $p$ | −43.05 | −22.03 | −19.81 | | | | | |
| | | −43.24 | −22.15 | −19.96 | | | | | |
| $^{17}$C | Def. 1, $n$ | −39.61 | −21.34 | −16.46 | −10.16 | −5.08 | −2.92 | | |
| | | −39.86 | −21.85 | −16.75 | −10.28 | −5.14 | | | |
| | Def. 1, $p$ | −45.68 | −27.60 | −21.81 | | | | | |
| | | −45.93 | −28.12 | −22.13 | | | | | |
| | Def. 2, $n$ | −40.00 | −21.76 | −16.81 | −9.89 | −5.29 | −3.27 | | |
| | | −40.25 | −22.27 | −17.11 | −10.01 | −5.35 | | | |
| | Def. 2, $p$ | −45.39 | −27.25 | −21.54 | | | | | |
| | | −45.65 | −27.78 | −21.86 | | | | | |
| $^{19}$C | Def. 1, $n$ | −40.06 | −22.13 | −17.30 | −11.22 | −5.74 | −3.52 | −1.03 | |
| | | −39.68 | −21.90 | −17.18 | −10.84 | −5.68 | −3.48 | | |
| | Def. 1, $p$ | −48.31 | −30.63 | −24.66 | | | | | |
| | | −47.94 | −30.40 | −24.53 | | | | | |
| | Def. 2, $n$ | −40.16 | −21.90 | −17.40 | −11.26 | −5.74 | −3.58 | −0.97 | |
| | | −39.79 | −22.13 | −17.29 | −10.88 | −5.67 | −3.55 | | |
| | Def. 2, $p$ | −48.24 | −30.58 | −24.60 | | | | | |
| | | −47.88 | −30.36 | −24.49 | | | | | |
| $^{21}$C | Def. 1, $n$ | −40.84 | −20.98 | −19.17 | −12.79 | −4.60 | −3.79 | −2.68 | |
| | | −41.28 | −21.14 | −19.27 | −13.28 | −4.64 | −3.80 | −2.60 | −2.68 |
| | Def. 1, $p$ | −51.28 | −31.23 | −29.55 | | | | | |
| | | −51.68 | −31.39 | −29.65 | | | | | |
| | Def. 2, $n$ | −40.92 | −20.78 | −19.31 | −12.98 | −4.50 | −3.80 | | −2.78 |
| | | −41.35 | −20.93 | −19.41 | −13.49 | −4.54 | −3.80 | −2.59 | −2.78 |
| | Def. 2, $p$ | −51.20 | −31.27 | −29.54 | | | | | |
| | | −51.59 | −31.44 | −29.64 | | | | | |

To explore the possible halos in odd-$A$ nuclei, we calculate the valence neutron and neutron matter radii. The results are given in Table 6 for cases with and without the pion. The pion contribution to the density distribution near the drip line is very small, and is also quite limited for the distribution of valence nucleon near closed-shell nuclei. Just judging by the difference between the valence neutron and neutron matter radii, we may find from Table 6 that $^{17}$C, $^{19}$C, and $^{21}$C are all the possible candidates of halo nuclei since they have the large valence neutron radius. However, the large valence neutron matter radius is not sufficient to be the definite evidence of halo formation, since the valence neutron matter radius is naturally large as the nucleus is near the drip line. According to Tables 3 and 4, the abnormal decrease of both the pion and spatial omega field energies only exists for $^{19}$C, which may serve as a referential evidence of halo formation. The abnormal decrease of the spatial omega field energy exists also for $^{15}$C, which shows that $^{15}$C is a possible candidate of halo nucleus. The spatial omega and pion fields in $^{17}$C do not decrease abnormally, but the abnormal rise of the spatial omega is observed. This helps exclude $^{17}$C as a halo nucleus in deformed RMF considering the referential evidence, although $^{17}$C may have a thick neutron skin. In Refs. [25] $\sim$ [27], the analyses based on the experiments show that there is no halo structure in $^{17}$C. Our conclusion on the exotic structure of $^{17}$C is consistent with those analyses. The valence neutrons of $^{15}$C and $^{19}$C occupy the levels $(1/2)^+$, which are from the spherical levels $1d$ 5/2 and $2s$ 1/2, respectively. The ground-state spin of $^{17}$C is $(3/2)^+$, given in Table 5. The spin and parity of $^{17}$C and $^{19}$C obtained here are consistent with experimental ones.[28] In the spherical case, the valence neutron of $^{19}$C occupies $1d$ 5/2, whereas the present occupation $(1/2)^+$ (from $2s$ 1/2) is due to the deformation, as pointed out in Ref. [22]. The halo formation in odd-$A$ carbon isotopes is closely related to the valence neutron occupation outside the closed shell. This is explicit for $^{21}$C. For $^{21}$C, the valence neutron with NL-SH occupies the level $(1/2)^+$ (from the spherical level $2s$ 1/2). Since the pion and spatial omega field energies with NL-SH are abnormally small (see Table 5), it is a halo nucleus according to the above discussions. However, the abnormal shift of $^{21}$C in NL3 does not occur for the pion and spatial omega field energies, seen in Table 4. In NL3, the valence neutron occupies the level $(5/2)^+$.

**Table 6** The valence neutron matter rms radii $r_{v,n}$ and matter rms radii of neutron $r_n$ for odd-$A$ nuclei of carbon isotope. Cases of Def. 1 and Def. 2 with NL-SH are given.

| | | $^{11}$C | $^{13}$C | $^{15}$C | $^{17}$C | $^{19}$C | $^{21}$C |
|---|---|---|---|---|---|---|---|
| Def. 1 | $r_{v,n}$ | 2.484 | 2.600 | 2.928 | 3.569 | 3.631 | 3.685 |
| | $r_n$ | 2.266 | 2.440 | 2.713 | 2.930 | 3.140 | 3.210 |
| Def. 2 | $r_{v,n}$ | 2.490 | 2.789 | 2.925 | 3.543 | 3.625 | 3.673 |
| | $r_n$ | 2.263 | 2.439 | 2.708 | 2.922 | 3.139 | 3.210 |

At last, we mention that the parameter set NL3 and NL-SH can give consistent descriptions for most of nuclei in carbon isotopes. For nucleus near the drip line (for instance, $^{21}$C), the theoretical results seem to be model dependent. However, this is a well-known but not-well-solved problem for the effective nuclear models, as they are applied to the nuclei closed to the drip line.

## 4 Summary

We have taken into account the pion meson in RMF for odd-$A$ nuclei. The spatial component of $\omega$-meson, associated with the non-zero vector current, is also included in the study. The equations of motion for nucleons and mesons are both solved in the method of expansion in the deformed harmonic-oscillator basis. The odd-$A$ nuclei of carbon isotopes have been investigated. The pion and spatial omega provide small attractions, which give rise to considerable influences on the single-particle energies rather than the bulk properties such as total binding energies, rms radii, and deformations. Numerical results are performed with NL-SH and NL3 parameter sets.

Due to the coupling between the valence neutron and the core, the core is polarized and the $\pm\Omega$ degeneracy is broken in deformed odd-$A$ nuclei. The $\pm\Omega$ splittings are large in $^{11}$C and $^{13}$C and go down as the isospin of odd-$A$ carbon isotopes rises. The large $\pm\Omega$ splittings in $^{11}$C and $^{13}$C are ascribed to the strong core polarizations due to the hole in the closed sub-shell $N = 6$ and shell $N = 8$, respectively.

Though the pion has a long interaction range, its role on isotopes that are close to the drip line or possible candidates of halo nuclei is abnormally smaller than expected before, and that can be explained by the structure of the pion source terms in deformed RMF. Due to the property of an isovector, the pion can shift slightly the relative potential depth of neutron and proton, contrary to the role of the rho meson. The binding energy of the valence neutron for carbon isotopes near the $\beta$-stability can be considerably shifted by the pion. The pion contribution is almost independent of the $\pm\Omega$ splitting. Based on the specific property of pion meson, the abnormal shift of pion field may serve as a referential evidence of halo formation, and so can the abnormal shift of the spatial omega field. According to these referential evidences, we may see that $^{19}$C has an explicit halo, $^{15}$C is a possible candidate of halo nucleus, and the halo in $^{21}$C is model-dependent.

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## Appendix: Formulations of Pion and Spatial Components of Vector Mesons

In this appendix, we will give the corresponding formulations for the pion in the cylindrical frame. The Dirac equations in co-ordinate space, pion contributions in the Dirac equations and pion field energy density are correspondingly given. The contributions of spatial components of vector mesons in the Dirac equations are also given.

In axially symmetric nuclei, the cylindrical frame is used,

$$x = r_\perp \cos\varphi, \quad y = r_\perp \sin\varphi, \quad z = z\,. \tag{A1}$$

The spinor $\psi_i$ with index $i$ is characterized by the quantum numbers, i.e., the total spin projection, parity, and iso-spin $\Omega_i$, $\pi_i$, and $t_i$. The Dirac equation with the vector potential $V$ and effective mass $M_N^*$ can be expressed in the form

$$(-\mathrm{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta M_N^* + V)\psi_i = \epsilon_i \psi_i\,. \tag{A2}$$

Using the expansion of $\psi_i$,

$$\psi_i(\boldsymbol{r}) = \frac{1}{2\pi}\begin{pmatrix} f_i^+(z,r_\perp)\,\mathrm{e}^{\mathrm{i}(\Omega_i-1/2)\varphi} \\ f_i^-(z,r_\perp)\,\mathrm{e}^{\mathrm{i}(\Omega_i+1/2)\varphi} \\ \mathrm{i}g_i^+(z,r_\perp)\,\mathrm{e}^{\mathrm{i}(\Omega_i-1/2)\varphi} \\ \mathrm{i}g_i^-(z,r_\perp)\,\mathrm{e}^{\mathrm{i}(\Omega_i+1/2)\varphi} \end{pmatrix}, \tag{A3}$$

Equation (A2) can be written explicitly as

$$\begin{pmatrix} M_N^*+V & 0 & \partial_z & \partial_\perp + \dfrac{\Omega_i+1/2}{r_\perp} \\ 0 & M_N^*+V & \partial_\perp - \dfrac{\Omega_i-1/2}{r_\perp} & -\partial_z \\ \partial_z & \partial_\perp + \dfrac{\Omega_i+1/2}{r_\perp} & M_N^*-V & 0 \\ \partial_\perp - \dfrac{\Omega-1/2}{r_\perp} & -\partial_z & 0 & M_N^*-V \end{pmatrix}\begin{pmatrix} f_i^+ \\ f_i^- \\ g_i^+ \\ g_i^- \end{pmatrix} = \epsilon_i \begin{pmatrix} f_i^+ \\ f_i^- \\ -g_i^+ \\ -g_i^- \end{pmatrix}. \tag{A4}$$

For $\sigma$ meson and the temporal component of vector mesons, the contributions in the Dirac equation and energy density are well known.[4−13] The following only gives the procedure including the pion and spatial component of vector meson in the Dirac equation and the energy density. The system Hamiltonian is given by

$$h = \sum p\dot{q} - \mathcal{L} = \bar{\psi}\mathrm{i}\gamma_0\partial^0\psi - \mathcal{L} \equiv h_1 + h_2\,, \tag{A5}$$

where $h_1$ is the Hamiltonian that includes all the parts except for pion and spatial components of vector mesons. For simplicity, we just write down $h_2$,

$$h_2 = \frac{f_\pi}{2m_\pi}\bar{\psi}\gamma_5\gamma_i\tau_3\nabla_i\pi_0\psi - \frac{1}{2}g_\omega\bar{\psi}\gamma_i\omega_i\psi - \frac{1}{2}g_\rho\bar{\psi}\gamma_i b_{0i}\tau_3\psi\,, \tag{A6}$$

where the pion is independent of time. The explicit expressions of $\pi_0$, $\omega_i$, and $b_{0i}$ are given as

$$\pi_0(y) = \frac{f_\pi}{m_\pi}\int \mathrm{d}^4x D_R^\pi(y-x)\nabla_i(\bar{\psi}(x)\gamma_5\gamma_i\tau_3\psi(x)) = -\frac{f_\pi}{m_\pi}\int \mathrm{d}^4x D_R^\pi(y-x)\boldsymbol{\nabla}\cdot(\psi^\dagger(x)\boldsymbol{\Sigma}\,\tau_3\psi(x))\,, \tag{A7}$$

$$\omega_i(y) = g_\omega\int \mathrm{d}^4x D_R^\omega(y-x)\bar{\psi}(x)\gamma_i\psi(x) = g_\omega\int \mathrm{d}^4x D_R^\omega(y-x)\psi^\dagger(x)\alpha_i\psi(x)\,, \tag{A8}$$

$$b_{0i}(y) = g_\rho\int \mathrm{d}^4x D_R^\rho(y-x)\psi^\dagger(x)\alpha_i\tau_3\psi(x) \tag{A9}$$

with $D_R^m$, $(m=\pi,\omega,\rho)$ being the retarded Green functions of mesons. The system energy operator related to $h_2$ is

$$\begin{aligned} H_2 = \int \mathrm{d}^3x h_2 = \int \mathrm{d}^3x_1\mathrm{d}^4x_2\Big\{&\frac{f_\pi^2}{2m_\pi^2}\bar{\psi}(x_1)\gamma_5(1)\gamma_i(1)\tau_3(1)\nabla_{i1}(D_R^\pi\nabla_{i2}[\bar{\psi}(x_2)\gamma_5(2)\gamma_i(2)\tau_3(2)\psi(x_2)])\psi(x_1) \\ &- \frac{1}{2}g_\omega^2\bar{\psi}(x_1)\gamma_i(1)D_R^\omega\bar{\psi}(x_2)\gamma_i(2)\psi(x_2)\psi(x_1) - \frac{1}{2}g_\rho^2\bar{\psi}(x_1)\gamma_i(1)\tau_3(1)D_R^\rho\bar{\psi}(x_2)\gamma_i(2)\tau(3)\psi(x_2)\psi(x_1)\Big\} \\ = \sum_{i_1,i_2}\int \mathrm{d}^3x_1\mathrm{d}^3x_2\Big[&-\frac{f_\pi^2}{2m_\pi^2}D_R^\pi(\vec{r})\Big\{\partial_{z1}(\psi^\dagger(\vec{x}_1)\Sigma_3\tau_3\psi(\vec{x}_1)) \\ &+\Big(\nabla_{r_\perp 1} - 2\frac{\Sigma_3 l_3}{r_\perp}\Big)(\psi^\dagger(\vec{x}_1)\Sigma_{r_\perp}(1)\tau_3\psi(\vec{x}_1))\Big\}_1 \times\{\cdots\}_2 \\ &-\frac{1}{2}g_\omega^2\psi^\dagger(x_1)\alpha_\varphi(1)D_R^\omega(\vec{r})\psi^\dagger(\vec{x}_2)\alpha_\varphi(2)\psi(\vec{x}_2)\psi(\vec{x}_1) \end{aligned} \tag{A10}$$

$$-\frac{1}{2}g_\rho^2\psi^\dagger(\vec{x}_1)\alpha_\varphi(1)\tau_3(1)D_R^\rho(\vec{r})\psi^\dagger(\vec{x}_2)\alpha_\varphi(2)\tau(3)\psi(\vec{x}_2)\psi(\vec{x}_1)\Big], \tag{A11}$$

where $\{\cdots\}_2$ means that the content in the bracket is the same as in $\{\cdots\}_1$ but with the index 2, and $r = |\vec{x}_1 - \vec{x}_2|$.

In order to obtain the Dirac equation, we need make the variation of system Hamiltonian $H$ $(= H_1 + H_2)$ over the Dirac spinors. The following relation is used for obtaining the Dirac equations

$$\delta\langle\bar{\psi}|H|\psi\rangle = E\delta N = 0\,, \tag{A12}$$

where $N$ is the nucleon number operator and $E$ is the eigen-energy matrix. In fact, the Dirac equations are obtained through setting coefficients of $\delta f^+$, $\delta f^-$, $\delta g^+$, $\delta g^-$ in Eq. (30) to be zero. Since the procedure is well known and straightforward, we omit the tedious details here. The final Dirac equation is

$$\begin{pmatrix} M^*+V+V_{\pi z} & V_{\pi\perp}+\dfrac{V_{\pi 0}}{r_\perp} & \partial_z & \partial_\perp^+ + \dfrac{\Omega_i+1/2}{r_\perp} \\ V_{\pi\perp}+\dfrac{V_{\pi 0}}{r_\perp} & M^*+V-V_{\pi z} & \partial_\perp^- - \dfrac{\Omega_i-1/2}{r_\perp} & -\partial_z \\ \partial_z & \partial_\perp^+ + \dfrac{\Omega_i+1/2}{r_\perp} & M^*-V-V_{\pi z} & -(V_{\pi\perp}+\dfrac{V_{\pi 0}}{r_\perp}) \\ \partial_\perp^- - \dfrac{\Omega-1/2}{r_\perp} & -\partial_z & -(V_{\pi\perp}+\dfrac{V_{\pi 0}}{r_\perp}) & M^*-V+V_{\pi z} \end{pmatrix} \begin{pmatrix} f_i^+ \\ f_i^- \\ g_i^+ \\ g_i^- \end{pmatrix} = \epsilon_i \begin{pmatrix} f_i^+ \\ f_i^- \\ -g_i^+ \\ -g_i^- \end{pmatrix}, \tag{A13}$$

where $\partial_\perp^\pm = \partial_\perp \pm V_{v\perp}$ with

$$V_{v\perp} = V_{\omega_\varphi} + V_{b_\varphi}\,, \tag{A14}$$

and

$$V_{\omega_\varphi} = g_\omega^2 \sum_{i_1} \int \frac{\mathrm{d}^3x_1}{2\pi} D_R^\omega(\vec{r})[f_{i1}^+ g_{i1}^- - f_{i1}^- g_{i1}^+]\,, \tag{A15}$$

$$V_{b_\varphi} = g_\rho^2 \sum_{i_1,t_3} t_3 \int \frac{\mathrm{d}^3x_1}{2\pi} D_R^\rho(\vec{r})[f_{i1}^+ g_{i1}^- - f_{i1}^- g_{i1}^+]\,, \tag{A16}$$

where the spatial components of vector mesons have the non-zero contribution from the angular part only. Therefore, the non-zero nucleon current in deformed nuclei is only in the plane perpendicular to the symmetric $z$-axis. The potentials from the pion meson are given by

$$V_{\pi z} = \frac{1}{2}\sum_i \frac{2f_\pi^2}{m_\pi^2} t_3 \partial_z \int \mathrm{d}^3x D_R^\pi(\vec{r})\mathcal{F}\,, \tag{A17}$$

$$V_{\pi\perp} = \frac{1}{2}\sum_i \frac{2f_\pi^2}{m_\pi^2} t_3 \frac{\partial_\perp}{r_\perp} \int \mathrm{d}^3x (r_\perp D_R^\pi(\vec{r}))\mathcal{F} \tag{A18}$$

$$V_{\pi 0} = -\frac{1}{2}\sum_i \frac{2f_\pi^2}{m_\pi^2} t_3 \int \mathrm{d}^3x 2D_R^\pi(\vec{r})\mathcal{F}\,, \tag{A19}$$

and $\mathcal{F}$ is

$$\mathcal{F} = \nabla_{r_\perp}\rho_{\pi_\perp} + \frac{\rho_{\pi_\perp}}{r_\perp} + \partial_z\rho_{\pi_z}\,, \tag{A20}$$

where

$$\begin{aligned} \nabla_{r_\perp}\rho_{\pi_\perp} + \frac{\rho_{\pi_\perp}}{r_\perp} &= \sum_i \Big(\nabla_{r_\perp} - 2\frac{\Sigma_3 l_3}{r_\perp}\Big)(\psi^\dagger(\boldsymbol{r})\Sigma_{r_\perp}\tau_3\psi(\boldsymbol{r})) \\ &= \nabla_{r_\perp}\sum_{i,t_3}\frac{t_3}{\pi}[f_i^+ f_i^- + g_i^+ g_i^-] + \sum_{i,t_3}\frac{t_3}{\pi r_\perp}[f_i^+ f_i^- + g_i^+ g_i^-]\,, \end{aligned} \tag{A21}$$

$$\partial_z\rho_{\pi_z} = \sum_i \partial_z(\psi^\dagger(\boldsymbol{r})\Sigma_3\tau_3\psi(\boldsymbol{r})) = \partial_z \sum_{i,t_3}\frac{t_3}{2\pi}[f_i^{2+} - f_i^{2-} + g_i^{2+} - g_i^{2-}]\,. \tag{A22}$$

Now we can show on what conditions the pion and spatial components of vector meson have non-zero contribution. For the even-even nuclei, it has the time reversal symmetry. So, if there is the Dirac spinor with positive $\Omega_i$,

$$\psi_i \equiv \{f_i^+,\ f_i^-,\ g_i^+,\ g_i^-\ \ \Omega_i\}\,, \tag{A23}$$

it has the time-reversed solution with the same energy

$$T\psi_i \equiv \{-f_i^-,\ f_i^+,\ g_i^-,\ -g_i^+\ \ -\Omega_i\}\,, \tag{A24}$$

where the time reversal operator is $T = \mathrm{i}\sigma_y K$ with $\sigma_y$ the Pauli matrix and $K$ the complex conjugation operator. We can easily find that the pion and spatial components of vector mesons will vanish for even-even nuclei where the spin is saturated by summing up states with both positive and negative $\Omega_i$'s. The non-zero contribution only appears for odd-$A$ nuclei that have one state ($\Omega_i$) where spin is not paired.

In order to solve the pion field $\pi_0$ in Eq. (A7) in the harmonic-oscillator basis, we write the source term

$$s_{\pi_0}(z,r_\perp) = -\frac{f_\pi}{m_\pi}\Big(\nabla_{r_\perp}\rho_{\pi_\perp} + \frac{\rho_{\pi_\perp}}{r_\perp} + \partial_z\rho_{\pi_z}\Big)\,. \tag{A25}$$

The corresponding potential $V_{\pi_0}$ is

$$V_{\pi_0}(z,r_\perp) = -\frac{1}{2}\frac{2f_\pi^2}{m_\pi^2}\int \mathrm{d}^3x D_R^\pi(\vec{r})\Big(\nabla_{r_\perp}\rho_{\pi_\perp} + \frac{\rho_{\pi_\perp}}{r_\perp} + \partial_z\rho_{\pi_z}\Big) = \frac{f_\pi}{m_\pi}\pi_0\,. \tag{A26}$$

After the solution $\pi_0$ to the Klein–Gorden equation is obtained in the basis expansion space, we return to the coordinate space by writing down the quantities such as fields, densities, and potentials in the harmonic-oscillator basis. The concrete procedure is similar to that in Ref. [15]. The potentials appearing in Eq. (A13) related to the pion meson are given as

$$V_{\pi z} = -t_3\partial_z V_{\pi_0}\,, \quad V_{\pi\perp} = -t_3\frac{\partial_{r_\perp}(V_{\pi_0}r_\perp)}{r_\perp} = t_3\Big(\frac{\partial V_{\pi_0}}{\partial r_\perp} + \frac{V_{\pi_0}}{r_\perp}\Big)\,, \quad V_{\pi 0} = 2t_3 V_{\pi_0}\,. \tag{A27}$$

For the energy of the pion field, it can refer to Eq. (A11) and is

$$E_\pi = -\langle\bar{\psi}|H_2|\psi\rangle_\pi = -\sum_{i_1,i_2}\int \mathrm{d}^3x_1\,\mathrm{d}^3x_2\Big[-\frac{f_\pi^2}{2m_\pi^2}D_R^\pi(\vec{r})\Big\{\partial_{z1}(\psi^\dagger(\vec{x_1})\Sigma_3\tau_3\psi(\vec{x_1}))$$
$$+\Big(\nabla_{r_{\perp 1}} - 2\frac{\Sigma_3 l_3}{r_\perp}\Big)(\psi^\dagger(\vec{x_1})\Sigma_{r_\perp}(1)\tau_3\psi(\vec{x_1}))\Big\}_1 \times \{\cdots\}_2\Big] = \frac{1}{2}\int 2\pi\,\mathrm{d}z r_\perp\,\mathrm{d}r_\perp \pi_0(x)s_{\pi_0}(x)\,. \tag{A28}$$

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