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LATTICE BOLTZMANN SIMULATION OF A SINGLE CHARGED ELLIPTIC CYLINDER IN A NEWTONIAN FLUID

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A two-dimensional dynamics model of an elliptic cylinder is derived by using the lattice Boltzmann method. With the present model, we have simulated the sedimentation of a single charged elliptic cylinder in a two-dimensional tube in a Newtonian fluid. Due to the polarizing effects and non-axial symmetry shape, there are the Coulomb force and the Coulomb torque on the elliptic cylinder during the sedimentation, which change its ordinary motion significantly. Comparing with the sedimentation of an un-charged elliptic cylinder under the same initial condition, we have further discussed the dynamics characteristics of the charged elliptic cylinder, and obtained some interesting results.

Keywords: Lattice Boltzmann method; charged elliptic cylinder; Newtonian fluid.

1. Introduction

Recently, the phenomena of tiny particle suspensions and their motion laws are of great interesting subjects.[1,2] For the suspensions of the tiny particles, there are many factors that decide their motion characteristics, and the charged statuses and shapes of the tiny particles are two important factors. Up to now, the tow and the three-dimensional sedimentation of the un-charged particles of different shapes, such as rectangle cylinder, circular cylinder, elliptic cylinder, sphere and ellipsoid of revolution etc. in a Newtonian and a non-Newtonian fluid have been simulated by using different methods, and the results agree with those from the experiments quite well,[3–9] but there are few reports about the researches of the charged particles. The suspensions of the charged particles occur in a variety of domains, such as the suspension of the charged particles in the air, the sedimentation and the aggregation of the red cells carrying negative charge,[10] and so on. Wan *et al.*[11]

simulated the suspension of a single charged circular cylinder in a Newtonian fluid by using the lattice Boltzmann method, which obtained some noticeable results. In its proceeding, an extra Coulomb force on the charged circular cylinder changed its motion characteristics significantly. As a more general case, the two-dimensional suspension of a charged elliptic cylinder in a Newtonian fluid has been simulated by using the lattice Boltzmann method in this paper. There is not only a Coulomb force, but also a Coulomb torque on the elliptic cylinder in our proceeding, they both add noticeable influences on the motion of the elliptic cylinder. By comparing with the motion of an un-charged elliptic cylinder under the same case, we have further discussed the dynamic characteristics of the charged elliptic cylinder, and obtained some interesting results.

2. Lattice Boltzmann Model

Let us consider an elliptic cylinder motion in the fluid within a vertical tube, the width of the tube is $L = 8a$, and the lengths of the major axis and the short axis of the elliptic cylinder are $2a$ and $2b$ respectively. During the sedimentation of the elliptic cylinder, the fluid in the tube satisfies the continuous equation and the Navier–Stockes equation.

If a 9-bit square model of two-dimension is adopted, the single particle contribution $f_i(\vec{x}, t)$ satisfies the following discrete lattice Boltzmann equation[12]:

$$f_i(\vec{x} + \vec{e}_i, t+1) - f_i(\vec{x}, t) = -\frac{1}{\tau}(f_i - f_i^{\mathrm{eq}}) \tag{1}$$

where $\vec{e}_i$ $(i = 0 \sim 8), f_i^{\mathrm{eq}}$ is the equilibrium distribution function and τ is the relaxation time. The equilibrium distribution function is taken as[12]

$$f_i^{\mathrm{eq}} = \alpha_i \rho \left[1 + 3\,\vec{e}_i \cdot \vec{u} + \frac{9}{2}(\vec{e}_i \cdot \vec{u})^2 - \frac{3}{2}u^2\right], \tag{2}$$

where

$$\alpha_0 = 4/9\,, \quad \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1/9\,,$$

and

$$\alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 1/36\,,$$

the density ρ and the macroscopic velocity $\vec{u}$ are defined by

$$\rho = \sum_i f_i\,, \quad \vec{u} = \sum_i f_i\,\vec{e}_i\,/\rho\,. \tag{3}$$

Using the Taylor expansion, the multi-scale technique and the Chapman–Enskog procedure, we can easily obtain the continuous equation and the Navier–Stockes equation:

$$\partial_t \rho + \partial_\alpha(\rho u_\alpha) = 0\,, \tag{4}$$

and

$$\partial_t(\rho u_\alpha) + \partial_\beta(\rho u_\alpha u_\beta) = \partial_\alpha p + \nu \partial_\beta[\rho(\partial_\alpha u_\beta + \partial_\beta u_\alpha)] \tag{5}$$

where p is the pressure, ν is the kinematic viscosity,and they are defined as follows

$$p = c_s^2 \rho\,, \quad \nu = \frac{(2\tau - 1)}{6}\,, \quad \text{and} \quad c_s^2 = \frac{1}{3}\,. \tag{6}$$

For the stationary and the dynamical boundaries, the methods proposed by Ref. 8 are employed. The hydrodynamic force on the elliptic cylinder can be evaluated based on the momentum exchange of the fluid and the solid boundaries.[8] The translation and the rotation of the elliptic cylinder are updated at each Newtonian dynamics time step by using a so-call half-step "leap-frog" scheme.

In the two-dimensional case, if a single linear charge is set in the fluid between two infinite planes, and the linear charge is parallel to the planes, the Coulomb force on it can by calculated by the mirror-image method and the iteration of the mirror-image[11]:

$$F_e = \frac{q}{2\pi\varepsilon_0\varepsilon_1} \sum_{i=1}^{\infty} q^{(i)} \left(\frac{1}{2(i-1)L + 2z_1} - \frac{1}{2(i-1)L + 2z_2} \right)\,, \tag{7}$$

where $\varepsilon_1, \varepsilon_2$ are the dielectric constants of the fluid and the plans respectively, q is the linear charge density, L is the distance between two infinite planes, z_1 and z_2 are the distances from the linear charge to the left plane and the right plane respectively, and

$$q^{(i)} = q \left(\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \right)^{2(i-1)+1}\,.$$

Thus the Coulomb force and the Coulomb torque on the elliptic cylinder can be obtained by a numerical integration procedure.

3. Simulation and Results

In this work, the fluid in the tube is a lubricating oil with a density 0.8 g/cm^3, and its dielectric constant and its kinematic viscosity are $\varepsilon_1 = 2.24$ and $\nu = 0.1$ cm^2/s respectively, while the walls are made of glass with a dielectric constant $\varepsilon_2 = 7.0$. The lengths of the major axis and the short axis of the elliptic cylinder are $2a = 0.1$ cm and $2b = 0.05$ cm respectively, its density is $\rho = 1.6$ g/cm^3. The x axis is horizontal to the right and $x = 0$ corresponds to the left wall of the tube and φ is the angle between the major axis of the elliptic cylinder and the horizontal. The sedimentation of the elliptic cylinder with a liner charge density $\sigma_e = 1.009364 \times 10^{-10}$ C/cm has been simulated.

In our simulation, the initial conditions are $x_0 = 0.186085L$ and $\varphi_0 = 0$, the Coulomb force is taken as the sum of the first 5 items of the Eq. (7),[11] the lattice size is $a = 13$ (lattice nodes), $\tau = 0.6$, and the inlet of the domain is always $20a$ from the moving particle, whereas the downstream boundary is $30a$ from the boundary.

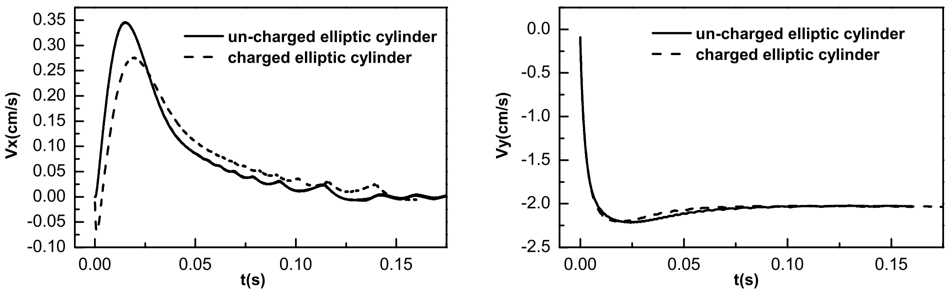

Fig. 1. The horizontal and the vertical velocities of the elliptic cylinder.

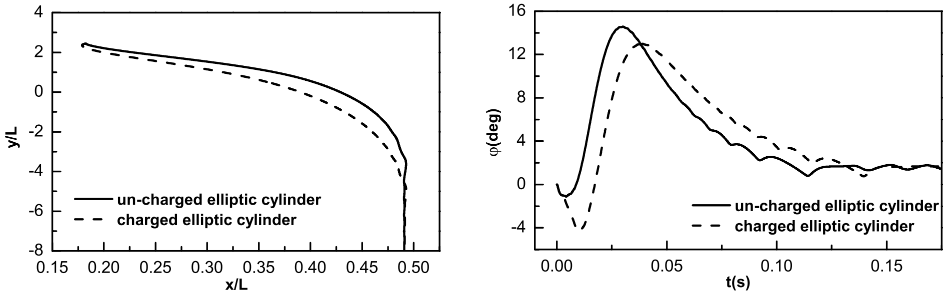

Fig. 2. The trajectories and the tilted angles of the elliptic cylinder.

Figure 1 shows us, unlike that of the un-charged one, the horizontal velocity of charged elliptic cylinder decreases first and then increases, which implicates that it shifts towards the left wall first and then goes back to the centerline. Because the dielectric constant of the liquid is small than that of the walls, there are attractive Coulomb forces between the charged elliptic cylinder and the walls (see Eq. (7)), which attract the charged elliptic cylinder left. From Fig. 2 we know that both the trajectory and φ of the charged elliptic cylinder are different from those of the un-charged one. Due to the non-axial symmetry shape, there are the Coulomb torques on the charged elliptic cylinder, which result in the difference of the time-dependent φ between the charged and the un-charged elliptic cylinders.

4. Conclusions

We have simulated the sedimentation of a single charged elliptic cylinder in a two-dimensional tube in a Newtonian fluid by using the the lattice Boltzmann method. Due to the polarizing effects and the non-axial symmetry shape, there are the Coulomb force and the corresponding torque on the charged elliptic cylinder during the sedimentation, which significantly change the horizontal translation and rotation of the elliptic cylinder.

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