

TWO-BODY CORRELATION CONTRIBUTIONS IN HALO NUCLEI WITH A RELATIVISTIC HARTREE APPROACH

W. Z. JIANG*, Z. Y. ZHU and W. Q. SHEN

Shanghai Institute of Nuclear Research, Chinese Academy of Sciences, Shanghai 201800, China
Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator,
Lanzhou 730000, China
**jiangwz02@hotmail.com*

X. J. QIU

Department of Physics, Shanghai University, Shanghai 200436, China

Z. Z. REN

Department of Physics, Nanjing University, Nanjing 210093, China

Received 30 September 2003

The relativistic density-dependent Hartree framework, where the relativistic two-body correlations are properly incorporated, is developed to study the properties of halo nuclei. The halo nucleon–meson vertex is reconstructed considering the nuclear potentials can be built dominantly from the two-body interactions. The two-neutron halo nucleus ^{11}Li , together with the one-neutron halo nucleus ^{19}C , is investigated. Separation energies, root-mean-square (rms) radii, and halo tails of above halo nuclei are nicely reproduced. The correlation contribution which provides essential attractions for halo neutrons is important to guarantee the relation $S_n > S_{2n}$ for ^{11}Li .

Keywords: Two-body correlation; halo nucleus; relativistic Hartree approach.

PACS Nos.: 21.60.Gv, 21.10.Dr, 21.60.Jz, 27.20.+n

The discoveries of the nuclear phenomena of halo nuclei near the drip lines may cast new light on the nuclear structure and nucleon–nucleon (N – N) interactions. For instance, in ^{11}Li ,¹ two neutrons of the last state are weakly-bound but different from the unbound two-neutron system. The large increase of the interaction cross section observed for the halo nucleus reaction is related to the large rms radius, as different from the empirical radii of mass dependence $1.2A^{1/3}$ (fm). Nowadays, experimental data for halo nuclei have been accumulated and the accuracy is improved. Meanwhile, new halo nuclei have been predicted theoretically.

Theoretical efforts^{2,3} based on various theoretical footings have achieved large success. In the few-body theory, the halo nucleus such as ^{11}Li is treated in a

three-body framework, where ^{11}Li is described by a $^9\text{Li} + n + n$ interacting system with different two-body potentials. In the many-body theory, one kind of work among a number of studies has investigated halo nuclei in the mean-field theory where nucleons of the weakly-bound state move in a renormalized potential having similar form but a different depth from that felt by the core nucleons.⁴ In the non-relativistic mean-field calculations, Bertsch *et al.*,⁵ and later Sagawa,⁶ reproduced some experimental data by introducing a factor to renormalize the potential felt by the weakly-bound neutrons. Similarly, in the spirit of the nonrelativistic scheme, an investigation with a phenomenological approach in RMF⁷ improved the results for halo nuclei and reproduced experimental data of neutron halos by modifying the potential for the weakly-bound nucleons of the last state (called as the halo nucleons later).

The interactions in ^{11}Li are strongly density-dependent in relevance to the diffusive matter distribution, and hence a description beyond the simple mean-field theory seems to be necessary. In the conventional shell model, pairing correlations included through the expansion of the oscillation basis up to high excitations have a much smaller effect on the ^{11}Li rms radius as expected.⁸ In Ref. 9, with pairing correlations treated in a particle number conservation method, some successful results are obtained, but a discrepancy of the separation energy still exists for halo nuclei such as ^{11}Li . Chinn *et al.*¹⁰ found that correlations play an important role in describing the large ^{11}Li radius with the input of the finite-range D1S effective interaction of Gogny, but such correlations do not fully account for the observed halo in ^{11}Li . In the framework of the Hartree–Bogoliubov theory, Meng *et al.* improved the description for ^{11}Li successfully.¹¹

The understanding of the properties of halo nuclei seems model-dependent. The effort has long been made towards a unique N – N interaction at low momentum.¹² Though this work has no relevance to the effort, it is of significance to investigate N – N interactions in the special laboratory, the halo nuclei. In this work we will not discuss the model dependence but try to understand properties of halo nuclei based on a realistic two-body force potential. Instead of studying the strongly density-dependent interactions in halo nuclei through pairing correlations, we investigate them by virtue of the two-body correlations obtained from the relativistic Brueckner–Hartree–Fock (RBHF) theory. We will apply the RBHF results¹³ to halo nucleus systems. Based on the realistic N – N interaction, it is also possible to explore some relevance between the many-body and few-body theories.

In the past, there are lots of works^{14–18} where the RBHF contributions are incorporated into the RMF potentials where the coupling constants are made density-dependent. The spherical nuclei and hypernuclei near β -stability have been investigated in these works, and it is found that two-body correlation contributions are important and favorable for the nuclear property description. Most recently, interactions in asymmetric matter with the RBHF approach have been discussed in Refs. 19 and 20 and some special halo nuclei have been investigated by the density-dependent RMF (e.g. Ref. 21). However, in this work we investigate properties of

halo nuclei mainly based on the understanding of the structure of N - N interactions in halo nuclei.

The correlations are conventionally understood in view of the fact that the N - N strong interaction has a repulsive core. In halo nuclei, the large spatial distribution of halo nucleons (or correspondingly the average large distance between halo and core nucleons) implies that the correlation contributions is negligible. Instead, the correlations among halo nucleons and core nucleons are important to describe properties for the core and the separation energy of halo neutrons.

The starting Lagrangian is in the following relativistic density-dependent Hartree approach (RDDH):

$$\begin{aligned} \mathcal{L}_{\text{RDDH}} = & \bar{\psi} \left[i\gamma_{\mu} \partial^{\mu} - M_N - g_{\sigma} \sigma - g_{\omega} \gamma_{\mu} \omega^{\mu} - g_{\rho} \gamma_{\mu} \tau_3 b_0^{\mu} + \frac{f_{\rho}}{2M_N} \sigma_{\mu\nu} \partial^{\nu} b_0^{\mu} \right. \\ & \left. - e \frac{1}{2} (1 + \tau_3) \gamma_{\mu} A^{\mu} \right] \psi + \frac{1}{2} (\partial_{\mu} \sigma \partial^{\mu} \sigma - m_{\sigma}^2 \sigma^2) \\ & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_{\omega}^2 \omega_{\mu} \omega^{\mu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} m_{\rho}^2 b_{0\mu} b_0^{\mu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu}, \quad (1) \end{aligned}$$

where ψ , σ , ω , b_0 are the fields of the nucleon, scalar, vector and neutral isovector-meson, respectively, with their masses M_N , m_{σ} , m_{ω} , m_{ρ} . A_{μ} and $A_{\mu\nu}$ are respectively the field of the photon and its field strength tensor. τ_3 is the isospin Pauli matrix of the third component. $F_{\mu\nu}$ and $B_{\mu\nu}$ are the strength tensors of ω and ρ mesons, respectively. g_i ($i = \sigma, \omega, \rho$) and f_{ρ} are the corresponding meson-nucleon couplings. Usually, g_{σ} and g_{ω} are density-dependent in RDDH, however, they may be density-independent in the case indicated later in the context.

The equation of motion for nucleons is

$$\begin{aligned} & \left(i\gamma_{\mu} \partial^{\mu} - M_N - g_{\sigma} \sigma - g_{\omega} \gamma_{\mu} \omega^{\mu} - g_{\rho} \gamma_{\mu} \tau_3 b_0^{\mu} \right. \\ & \left. + \frac{f_{\rho}}{2M_N} \sigma_{\mu\nu} \partial^{\nu} b_0^{\mu} + e \frac{1}{2} (1 + \tau_3) \gamma_{\mu} A^{\mu} + V_R \right) \psi = 0 \quad (2) \end{aligned}$$

here V_R is a potential induced by the requirement of Lorentz invariance of the Lagrangian and it is related to the derivative of the density-dependent coupling constants with respect to the local density,¹⁷

$$V_R = \frac{\partial g_{\omega}(\rho_i)}{\partial \rho_i} \rho_B \omega_{\mu} - \frac{\partial g_{\sigma}(\rho_i)}{\partial \rho_i} \rho_S \sigma, \quad i = c, h \quad (3)$$

where ρ_B is the total nucleon density, ρ_S is the total scalar density, and ρ_c , ρ_h are the nucleon densities of the core and halo, respectively. The density-dependent coupling constants are obtained from the RBHF potentials of Bonn A¹³ through the following relation¹⁴:

$$U_S(\rho) = -\frac{g_{\sigma}^2(\rho)}{m_{\sigma}^2} \rho_S, \quad U_V(\rho) = -\frac{g_{\omega}^2(\rho)}{m_{\omega}^2} \rho, \quad (4)$$

where U_S and U_V at the nuclear density ρ are the scalar and vector RBHF potentials, respectively.

The RBHF results for U_S and U_V in nuclear matter is not reliable at very low densities, and hence an appropriate extrapolation is needed to get the coupling constants at very low densities. At the zero density, U_S and U_V may be set to zero, as indicated in Eq. (4). The scalar and vector potentials are almost linearly dependent on the density in RMF at very low densities, since the contribution in terms of high powers of density to the potentials is negligible. Thus a polynomial fit of U_S and U_V with respect to density could remove the sensitivity of the extrapolation in obtaining coupling constants¹⁶ at very low densities. In particular, the cancellation of U_S and U_V in the Dirac equations leads to a more accurate result.

In principle, one also has to include, for instance, density-dependent effects for the rho meson–nucleon vertex. For this purpose one would need the microscopic self-energies of asymmetric matter, but such calculations are rather scarce and much more complicated. Most recent study of Ma *et al.*¹⁸ indicates that it has little density-dependence for the rho meson–nucleon vertex. For these reasons, we restrict ourselves to density-dependent coupling constants for the ω and σ mesons only, similar to the treatment in Ref. 19. The rho meson coupling constants are: $g_\rho = 3.03$ ($g_\rho^2/4\pi = 0.73$),²² $f_\rho/g_\rho = 6.6$.

The density-dependent coupling constants g_σ and g_ω for the core and the halo nucleons are determined in the following. The small separation energy of the last state nucleons of halo nuclei suggests that the core and halo particles are separately localized with a relatively large range between the core and halo nucleons. Therefore, the core and halo densities are respectively determined by the core and halo particle numbers. The density-dependent coupling constants for core nucleons are the following:

$$g_{\sigma,c} = g_\sigma(\rho_c), \quad g_{\omega,c} = g_\omega(\rho_c). \quad (5)$$

Referring to Eq. (4), a direct way to determine the halo nucleon vertex can be to construct the local potential for halo nucleons, and hence the coupling constant for halo nucleons can be expressed as $g_i^2 = \alpha g_{i,c}^2 + \beta g_{i,h}^2$, $i = \sigma, \omega$ with α, β two incoherent coefficients. In halo nuclei, the local density related to the potential for halo nucleons is small, and we make expansion for the potential and coupling constant at small densities. For the potential, it has $U(\rho) \simeq a\rho + b\rho^2$ by omitting high-order powers of density. According to Eq. (4), the coupling constant is given as

$$g \simeq c + d\rho \quad (6)$$

with a, b, c and d the expansion constants. It implies that the coupling constant for halo nucleons can be expressed by the linear combination of a constant and the coupling constant dependent on the halo density.

Since the correlation contribution is much weaker between the halo and core interaction, the interactions among halo nucleons and between the core and halo

nucleons have to be distinguished. Considering the single-nucleon potentials built dominantly from the two-body interactions, we may construct the single halo-nucleon potentials by performing the average over different numbers of two-body interactions among halo nucleons and between nucleons of the halo and the core. In the actual treatment, the correlation between the core and halo is reasonably neglected and density-independent empirical coupling constants are introduced to describe the halo-core interaction. The coupling constants for halo nucleons are thus given as follows. If there is only one nucleon in the halo, they just take the empirical (density-independent) values. For nuclei with two or more halo nucleons, the correlations among halo nucleons play an important role in the mean field, and the coupling constants of halo nucleons are then the mean values of empirical coupling constants and density-dependent ones, i.e.

$$g_{i,h} = \xi g_i + (1 - \xi)g_i(\rho_h), \quad i = \sigma, \omega \quad (7)$$

where ξ is a weight coefficient reflecting the interactions both among halo nucleons and between the core and the halo. Apparently, at a low density for halo the nucleon-meson vertex has the same form as that given in (6). For the halo nucleons, the number of the two-body interactions is $n_h = C_{N_h}^1 C_{N_c}^1 + C_{N_h}^2$ with N_h and N_c the particle numbers of the halo and the core, respectively. Then ξ is calculated by

$$\xi = \frac{C_{N_h}^1 C_{N_c}^1}{C_{N_h}^1 C_{N_c}^1 + C_{N_h}^2}, \quad (8)$$

where $C_{N_h}^2 = 0$ as $N_h < 2$. For ^{11}Li , $n_h = C_2^1 C_9^1 + 1 = 19$, $\xi = 0.947$ and for ^{19}C , $\xi = 1$. (5) and (7) imply that the surface of the mean field of the core has a large impact on the halo, whereas the effect of the halo particles on the mean field of the core is nearly negligible, as also mentioned in Refs. 10 and 23. Through coupling to the mean field, $g_{i,h}$ in (7) leads to locally density-independent and density-dependent potentials for halo nucleons.

Formulas (7) and (8) that elucidate the correlations among halo nucleons and the interactions between the core and the halo are basic in our calculation. The density-independent coupling constants in (7) are

$$g_\sigma = 9.01, \quad g_\omega = 11.54, \quad (9)$$

where g_ω is fixed, taking the value of $g_\omega(\mathbf{k}^2 = 0)$ in Ref. 13 to have the same starting footing as the coupling constants of density-dependence, while as the unique adjustable parameter in the model g_σ is determined by fitting the two-neutron separation energy of ^{11}Li and deviates from that in Ref. 13 by about 5%. Though the scalar coupling constant g_σ , mapping all essential attractions, is somehow an adjustable constant, it is in the vicinity of the empirical value. It seems an improvement that we just need to adjust one parameter g_σ other than two adjustable parameters in Ref. 7.

Now we may perform the calculation for a concrete halo nucleus ^{11}Li after details of the theoretical framework are determined. In a self-consistent mean-field

calculation, the filling of nucleons on energy levels for halo nuclei is the same as that for normal nuclei. The last state of ^{11}Li is $1p_{1/2}$. The binding energies of the last state of ^{11}Li is about 0.71 MeV, which is a small value close to the continuum limit. Compared with experimental data, the single-neutron separation energies (S_n), two-neutron separation energies (S_{2n}) and rms radius of ^{11}Li are well reproduced, as given in Table 1. The matter density distribution of ^{11}Li shown in Fig. 1 is in the scope determined by experimental data.⁴ To show the role of correlation in the halo, we make a comparison to the results obtained as ξ is set to be 1 (without the correlation contribution in the halo). A direct comparison is impossible since the S_n and S_{2n} are now minus as taken $\xi = 1$. In order to display the correlation contribution, we readjust the g_σ to fit the $S_n = 0.719$ MeV for ^{11}Li given in Table 1, and then take $\xi = 1$ to calculate the S_{2n} and rms radius. By doing so, we obtain $S_{2n} = 0.958$ MeV, and $r_{\text{rms}} = 3.00$ fm for ^{11}Li , which deviate from those extracted from experiments largely. This indicates that the correlation contribution for halo neutrons is important to guarantee the relation $S_n > S_{2n}$ in ^{11}Li as given by experiments. Moreover, since the correlation effects play the role of an attraction, the neglect of the correlations in the halo requires the attraction to be compensated

Table 1. The halo neutron separation energies (in MeV) and the rms radii (in fm) of ^{11}Li and ^{19}C . Main references for experimental data are denoted in the table.

Nucleus	S_n	S_n^{expt}	S_{2n}	S_{2n}^{expt}	r_{rms}	$r_{\text{rms}}^{\text{expt}}$	Ref.
^{11}Li	0.719	0.730	0.296	0.295 ± 0.035	3.125	3.12 ± 0.30	24, 25
^{19}C	0.328	0.24 ± 0.1	3.224	4.350	3.190		24, 27
		0.53 ± 0.13				3.10 ± 0.1	30

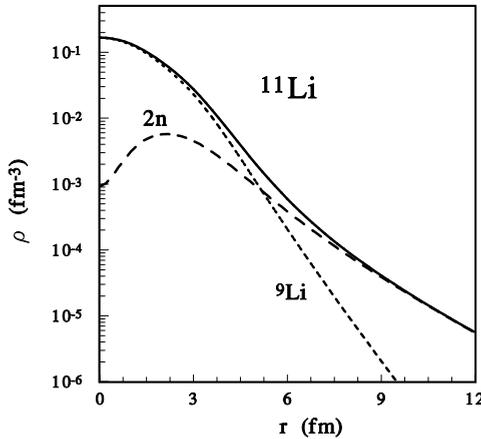


Fig. 1. Calculated matter density distributions of ^{11}Li . The solid line is for the density distribution of ^{11}Li . The short dashed and dotted lines represent the halo neutron and core ^9Li density distributions, respectively.

by the larger g_σ so as to reproduce the separation energies, and that leads to a much smaller rms matter radius. This also demonstrates the importance of correlations in the description for halo nuclei.

The experimental S_n of ^{11}Li depends on whether the last neutron of ^{10}Li occupies $s_{1/2}$ or $p_{1/2}$ orbital measured in experiments. This experimental uncertainty is discussed in a lot of references (for reviews, see Refs. 25 and 26), and the $s_{1/2}$ occupation leads to a smaller S_n of ^{11}Li . In our calculation, the last neutron of ^{10}Li occupies the $p_{1/2}$ orbital naturally, which gives the S_n of ^{11}Li compatible with the related experimental values.^{24,25}

^{19}C is another typical halo nucleus. Since the observation of halo structure in ^{19}C ,^{27,28} lots of efforts have contributed to further studying. In Ref. 29, the probable structure of ^{19}C is analyzed through the core polarization. Though the present model can probably incorporate some phenomenological effects of core polarizations through the density-dependent interactions, it does not consider the dynamical core polarization. In the following we just perform a simple calculation for halo nucleus ^{19}C with the free parameter given in (9).

The nucleon occupation order in ^{19}C is similar to that in normal nuclei. The calculation indicates that the halo neutron is not bound if the halo neutron in ^{19}C is on the level $1d_{5/2}$ according to its normal order. The system energy of ^{19}C is lowered, as the halo nucleon is on the level $2s_{1/2}$. The halo neutron in ^{19}C can thus be naturally on $2s_{1/2}$, which is consistent with the awareness that halo nucleons prefer the small orbital number due to the centrifugal barrier. The small binding-energy 0.60 MeV for the last state of ^{19}C is also very close to the continuum limit. The obtained rms radius is within experimental errors. The S_n is close to the experimental value. Theoretical results are also given in Table 1. It is found by a microscopic cluster model in Ref. 31 that a $1/2^+$ assignment for ^{19}C which presents a dominant $^{18}\text{C}(0^+) + n$ structure is consistent with the result of Ref. 30. Based on a structure of the $^{18}\text{C}(0^+)$ plus a halo neutron of $2s_{1/2}$ state our theoretical results for ^{19}C fit the data fairly well, which seems to be coincidentally consistent with the analyses of Ref. 31. The matter density distribution of ^{19}C is plotted in Fig. 2, and the halo tail is well reproduced. The rms radius of ^{19}C is in good agreement with that extracted from experimental data.

In the present calculation, the binding energy of ^{11}Li has a small deviation (about 2.6 MeV) from the experimental value, but the separation energy calculation is accurate because of the cancellation of the deviations between different neighboring nuclei. For ^{19}C , the S_{2n} has a not-so-small deviation from the experimental result. The reason for that might be attributed to the microscopic core polarization, while a microscopic treatment of core polarizations is beyond the present investigation.

It would be interesting to make a brief discussion on the above treatment for halo nuclei. Two very different models are widely used for the study of halo nuclei. One is the few-body model which treats the halo nucleus ^{11}Li as a core-neutron-neutron framework with the point-like and structureless core. Another model is

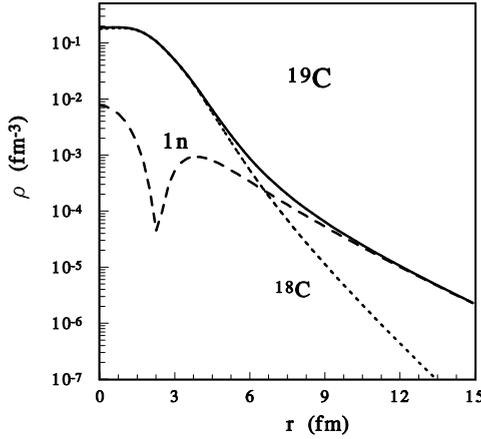


Fig. 2. The same as shown in Fig. 1, but for ^{19}C .

the self-consistent mean-field model. It has been frequently applied to nuclei far from stability in recent years. In the mean-field model, one does not distinguish the nucleons in the core or the halo neutrons outside the core. The solution of the ground state is obtained from self-consistent iterations. However, it fails for the description of ^{11}Li due to the overbinding of the last-state neutrons. Here we have developed an effective mean-field model which distinguishes core or halo nucleons. This model includes the basic idea of the few-body model and it is also beyond the few-body model at the same time because the core has a structure now in the mean-field model. The model therefore involves the characteristics of the few-body and the many-body models, and may possibly bridge the gap between two kinds of models through further explorations.

In summary, we have analyzed two-body correlation contributions in halo nuclei in a relativistic density-dependent mean-field framework. The halo nucleon-meson vertex in the mean field is reconstructed through a physically simple relation obtained from counting the number of two-body interactions. The correlation contribution which provides essential attractions for halo neutrons is important to guarantee the relation $S_n > S_{2n}$ for ^{11}Li . It is interesting that some quantities of the one-neutron halo nucleus ^{19}C are reproduced in the same framework with the same parameter. The theoretical results indicate our consideration and understanding of N - N interactions in halo nuclei is simple but appropriate. Since we work in a one-body and mean-field framework, considerations of some dynamic factors such as pairing correlations and core excitations would not be enough.

Acknowledgments

One of authors W.Z.J. would like to thank Dr. D. Q. Fang for his careful reading of the manuscript and nice discussion. This work is supported in part by the Major State Basic Research Development Program in China under Contract

No. G200077400, Chinese Academy of Sciences Knowledge Innovation Project No. KJCX2-N11, and fund of China Scholarship Council.

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