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LATTICE BOLTZMANN SIMULATION OF DEFORMABLE MEMBRANE IN FLUID

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A lattice Boltzmann method is proposed to simulate the two-dimensional membrane. Numerical simulation shows that at a critical value of membrane tension, the pattern of the membrane at transient state behaves like a standing wave with a node staying at rest. In addition, when the membrane is relatively soft or stiff, it will evolve into a steady-state close to its initial straight pattern.

1. Introduction

The complicated interaction of the deformable membrane with the surrounding fluid flow is not well understood. The membrane exerts force on the fluids through its inertia and elastic response and deforms according to the hydrodynamic force acting on it. The examples in the nature are the blood vessels, swimming fish and flapping flags. In this paper, we proposed a lattice Boltzmann method to simulate a deformable membrane. The simulation shows that the steady-state of the membrane is close to its initial straight pattern when the membrane is soft or stiff. At a critical value for the tension, pattern of the membrane at transient state behaves like a standing wave with a node staying at rest.

2. Lattice Boltzmann method and Boundary condition

The lattice Boltzmann equation reads[1,2]

$$f_i(\mathbf{x}+\mathbf{e}_i, t+1) - f_i(\mathbf{x}, t) = -\frac{1}{\tau}(f_i - f_i^{eq}), \tag{1}$$

where τ is a constant and the equilibrium distribution functions are of the form

$$f_i^{eq} = \alpha_i \rho \left[1 + 3\mathbf{e}_i \cdot \mathbf{u} + \frac{9}{2}(\mathbf{e}_i \cdot \mathbf{u})^2 - \frac{3}{2}u^2\right]. \tag{2}$$

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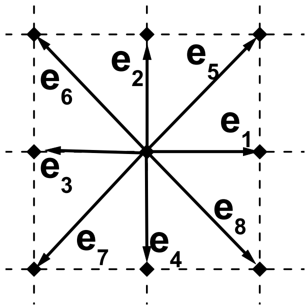

Fig. 1. Basic cell for the two-dimensional "nine-speed" lattice Boltzmann model.

In the model on a square lattice in two dimensions,

$$\mathbf{e}_0 = (0,0)\,,$$
$$\mathbf{e}_i = (\cos\pi(i-1)/2\,,\sin\pi(i-1)/2)\,,\qquad i=1,2,3,4\,,$$
$$\mathbf{e}_i = (\cos\pi(2i-1)/4,\sin\pi(2i-1)/4)\,,\qquad \text{for } i=5,6,7,8\,.$$

are the nine possible velocity vectors as shown in Fig. 1. $\alpha_0 = 4/9$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1/9$, and $\alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 1/36$. The density ρ and the velocity $\mathbf{u}$ are defined by

$$\rho = \sum_i f_i, \qquad \mathbf{u} = \sum_i f_i\mathbf{e}_i/\rho. \tag{3}$$

The macroscopic equations can be obtained by a Chapman-Enskog procedure. The viscosity in the macroscopic equations is

$$\nu = \frac{(2\tau - 1)}{6}.$$

The non-slip boundary condition proposed by Fillipova[3] is applied at solid-fluid boundary. The momentum exchange in the solid-fluid interface determines the force acting on the solid boundary.[4] The accuracy of this lattice Boltzmann scheme has been demonstrated by simulating the sedimentation of a circular cylinder in a vertical tube and comparing the simulation results with those obtained from a second-order finite-element scheme.[4]

3. Membrane

A piece of membrane is discretized into segments. Each segment is simplified to a mass particle and connected to its neighbors by straight lines. The mass particle can only move vertically and its dynamical equation could be[5]

$$F\delta x + T\tan\alpha_2 + T\tan\alpha_1 = \delta x m a, \tag{4}$$

where F is the hydrodynamic force exerting on the segment calculated by the momentum exchange between the the segment and fluid; δx is the length of every segment; T is the tension in the membrane which is assumed as a constant; m is the mass of a segment. $\tan\alpha_1$ and $\tan\alpha_2$ are the slopes of the straight lines connecting the adjacent mass particles; a is vertical acceleration of the segment. The motion of each mass particle is updated at each time step by using a so-called half-step "leap-frog" scheme.[6]

4. Simulation results

A straight membrane with length L=156 lattice units and two ends fixed is initially placed horizontally in the center of a box of 256×256 lattice nodes. In the simulation, the initial pattern of the membrane is a straight line parallel to x axis. The membrane is divided into L segments. Each segment has one lattice unit in the x direction. The mass of each segment is m=137. The density of fluid is $\rho_0 = 2.7$. The fluid velocity at inlet and outlet is always fixed to be $v_0 = 0.05$. $\tau = 0.524$. Non-slip boundary condition is applied on the top and bottom solid boundary. Initially, the membrane has a velocity distribution along vertical direction as:

$$\frac{dy}{dt} = -0.02 \sin \frac{\pi x}{L}. \tag{5}$$

This initial velocity distribution makes the membrane leaves the initial position which is a stable state for low fluid velocity at inlet and outlet.

In Fig. 2 we show the patterns of the membrane at different time for $T = 0.18, 0.2$ and 1.0 respectively. When T=0.18 and 1.0, the membrane reaches steady state after 64000 time-steps while it will take 564000 time-steps to arrive at the steady state for $T = 0.2$. The most interesting observation is that there exits a critical value about $T = 0.2$ at which the intermediate transient state of the membrane behaves like a standing waves with a fixed point (node) at $x \approx 112$. Fig. 3 displays the steady states for different values of tension. When $T < 0.2$, the final deformed membrane falls close its initial straight line, with one maximum displacement away from the initial position at each side. When $T \geq 0.3$, on the other hand, the entire membrane evolve to one side of its initial straight line, with only one maximum displacement. The membrane reaches it maximal displacement from its initial position at about $T = 0.3$. When T is very small or T is larger the steady state of the membrane is quite close to the initial position. In fact, there are three forces: hydrodynamic,

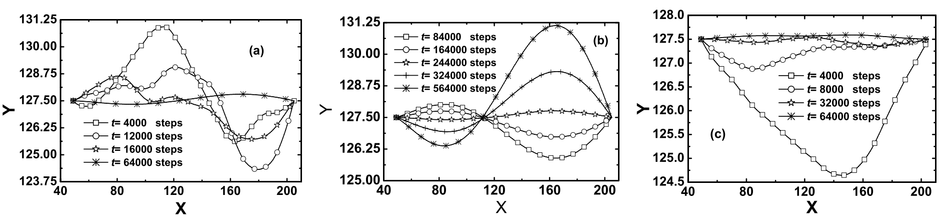

Fig. 2. Evolution of shape of the membrane for the cases of tension T=0.18(a), 0.20(b), 1.0(c). The last step of the shape is steady. Figs. (a) and (c) evolve to steady states much faster than (b).

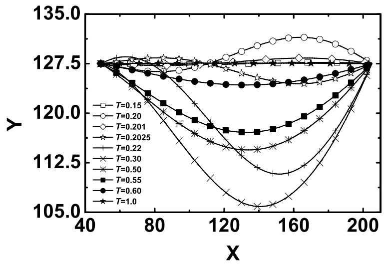

Fig. 3. The steady states of the membrane for different values of tension T.

shear stress and tension. When T is small, the membrane is soft, the shear stress plays the most important role which keeps the membrane almost straight. When T is large enough, the tension makes the membrane stay at its equilibrium initial straight state. Only at an intermediate value of T, will the membrane reach its maximal displacement.

Since the system is symmetric along $y = 127.5$, the asymmetric pattern shown in Fig. 2 implies that there should be another asymmetric solution. Our numerical simulation shows that the another asymmetric pattern can be found for initial velocity

$$\frac{dy}{dt} = 0.02 \sin \frac{\pi x}{L}. \tag{6}$$

In summary, we have proposed a lattice Boltzmann method to simulate the two-dimensional membrane. Numerical simulation shows that at a critical value of T, the transient state of the membrane behaves like a standing wave with a node at rest. When the membrane is relatively soft or stiff, the steady state of the membrane is close to its initial straight state.

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