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Collective Oscillation of Relativistic Electrons in Hot Plasma

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(Received 5 March 2003)

The interactions between relativistic electrons in a hot plasma are analyzed theoretically. By splitting the electron density fluctuations into the individual part and the collective part, we are concerned with the collective oscillation of the relativistic electrons resulting from the Coulomb interactions. Consequently, we derive the frequency of the hot plasma and the “Debye length” with relativistic modification.

PACS: 52.35.−g, 52.27.Ny

The advent of the chirped-pulse amplification (CPA) technique, coupled with the development of solid-state lasers capable of delivering ultra-short pulses, has opened the new field of ultrahigh-intensity physics.[1] The intense laser-plasma interaction has consequently received much theoretical and experimental attention.[2] However, even the intense laser-target interaction may produce hot or super-hot plasma, some studies still just took the dynamic parameters of the plasma as the non-relativistic ones. In addition, the electrons were taken as non-relativistic particles.[3–5] In fact, the interaction of the intense laser with a target can produce energetic particles, which even reach about MeV energy scale.[6] In this Letter, we attempt to take the relativistic effect into account.

The mass \( m \) of the hot plasma electron does not simply equal to the rest mass \( m_c \) of the electron. It satisfies the well-known relativistic relation: \( m = m_c/(1 - v^2/c^2)^{1/2} \). We should use relativistic theory to redefine the motion of hot plasma electrons. Therefore, in this work, we consider that the interaction between the relativistic electrons in hot plasma brings about organized behaviour and collective oscillation.

In principle, the field produced by relativistic electron should be described by the Lienard–Weichert potential (in Gauss unit):

\[
A(x, t) = \frac{ev}{c(r - \frac{v}{c} \cdot r)}, \quad \varphi(x, t) = \frac{e}{r - \frac{v}{c} \cdot r}.
\]

Usually, \( \varphi \) is rather important than \( A \). Therefore, we simply ignore the contribution from \( A \). At the same time, since angle function \( \frac{v}{c} \cos \theta \) (\( \theta \) is the angle between \( v \) and \( r \)) exists in \( \varphi \), the second term in the denominator is relatively small, compared with the first term. Thus, in order to simplify the calculation and to obtain the semi-quantitative result, we omit the second term and assume that the Coulomb interaction between electrons plays an essential role in the problem. In this case, the Coulomb potential between the \( i \)th and \( j \)th electrons expanded as a Fourier series in a box of unit volume with periodic boundary conditions is as follows:

\[
U(|x_i - x_j|) = -\frac{e^2}{|x_i - x_j|} = 4\pi e^2 \sum_k (1/k^2)e^{i k \cdot (x_i - x_j)},
\]

hence the Coulomb force will take the form:

\[
F_i = -\sum_j \frac{\partial U(|x_i - x_j|)}{\partial x_i} = -4\pi e^2 \sum_{j \neq i, k \neq 0} \frac{(k/k^2)e^{i k \cdot (x_i - x_j)}}{2}.
\]

On the other hand, we can write down the expression for the force acting on an electron within the framework of relativistic theory as follows:

\[
F = \frac{dP}{dt} = \frac{d}{dt}\left[m_c \sqrt{1 - v^2/c^2}\right] = m_c \left[\frac{d}{dt}\left(v + \frac{v}{c^2} \frac{d}{dt} (\frac{d}{dt} \cdot v)\right)\right] = m_c \gamma \left[\frac{d}{dt}v + \frac{v}{c^2} \frac{d}{dt} (\frac{d}{dt} \cdot v)\right] = m_c \gamma \left[\frac{d}{dt}v + \frac{v}{c^2} \frac{d}{dt} (\frac{d}{dt} \cdot v)\right] = m_c \gamma \left[\frac{d}{dt}v + \frac{v^2}{c^2} \frac{d}{dt} (\frac{d}{dt} \cdot v)\right],
\]

where \( \gamma \) is the Lorentz factor of the electron: \( \gamma = \left(1 - v^2/c^2\right)^{-1/2} \); \( F \) denotes the force acting on the electron, and \( m_c, v, \) and \( P \) denote the rest mass, the velocity and the momentum for this electron, respectively.

Here we should pay some attention to the third term on the right-hand side of Eq. (3):

\[
\left|\left(\frac{d}{dt} \cdot v\right) \times v\right| = \frac{dv}{dt} v^2 \sin \theta,
\]

where \( \theta \) is the angle between \( \frac{d}{dt} \cdot v \) and \( v \). Since the angle functions exist, the third term in this equation

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* Supported by the Knowledge Innovation Project of Chinese Academy of Sciences under Grant No KJCX2-SW-N02.
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is rather small, compared with the first two terms. Hence, we simply omit this term to avoid complicated calculation and only expect a semi-quantitative result. Thus, we have the expression for the acceleration in the following form:

\[
\dot{\mathbf{v}} = \frac{\mathbf{F}}{m_e \gamma (1 + \frac{v^2}{c^2} \gamma^2)} = \frac{\mathbf{F}}{m_e \gamma (1 + \beta^2 \gamma^2)},
\]

(4)

with Lorentz parameters \( \beta = \frac{v}{c}, \gamma = (1 - \beta^2)^{-1/2} \).

Equations (1)-(4) are important for us to study the behaviour of the interaction between relativistic electrons in hot plasma.

By virtue of Eqs. (2) and (4), we may take the following form for the equation of motion of the ith electron:

\[
\dot{\mathbf{v}}_i = - \left[ 4 \pi e^2 i/m_e \gamma_i (1 + \beta_i \gamma_i^2) \right] \cdot \sum_{j \neq i, k \neq 0} (k/k^2) e^{i k \cdot (x_i - x_j)}.
\]

(5)

Eq. (5) is very difficult to be solved. Many years ago, Pines et al.\textsuperscript{[7]} took some approach to deal with this kind of equation. Here we follow the similar approach but take into consideration of the relativistic effect, which is just mentioned above.

First, we assume that we are dealing with point particles, so that the particle density in the box of unit volume may be given as follows:

\[
\rho(x) = \sum_i \delta(x - x_i).
\]

(6)

It is more convenient to work with the Fourier components of the density, and they are given by

\[
\rho_k = \int \mathbf{d}x \rho(x) e^{-ik \cdot x} = \sum_i e^{-ik \cdot x_i},
\]

(7)

where \( \rho_0 \) represents the mean electron density \( n \), and \( \rho_k \) with \( k \neq 0 \) describes the fluctuations with respect to the mean density. In this case, Eq. (5) may be re-expressed as

\[
\dot{\mathbf{v}}_i = - \left[ 4 \pi e^2 i/m_e \gamma_i (1 + \beta_i \gamma_i^2) \right] \cdot \sum_{k \neq 0} (k/k^2) \rho_k e^{ik \cdot x_i}.
\]

(8)

Here we note that \( \rho_k \) determines the force acting on each particle.

By differentiating Eq. (7) with respect to the time \( t \) and using Eq. (5) for \( \dot{\mathbf{v}}_i \), we may obtain

\[
\ddot{\rho}_k = - \sum_i (\mathbf{k} \cdot \mathbf{v}_i)^2 e^{-ik \cdot x_i} \cdot \sum_{i, j, k \neq 0} \left[ 4 \pi e^2 i/m_e \gamma_i (1 + \beta_i \gamma_i^2) k^4 \right] k \cdot k' \cdot \exp[i(k' - k) \cdot x_i] e^{-ik' \cdot x_j}.
\]

(9)

The first term on the right-hand side of Eq. (9) arises simply from the random thermal motion of the individual electrons. The second term represents the effects of electrons interaction.

One may split the sum over \( k' \) into two parts. The first part, with \( k' = k \), makes the phase factors \( \exp(i(k' - k) \cdot x_i) \) equal 1. The second part, with \( k' \neq k \), contains phase factors \( \exp(i(k' - k) \cdot x_i) \). Since there are a very large number of particles distributed very nearly in random positions, these terms tend to average out to zero. As the first approximation, one may neglect such terms. With the random-phase approximation, we then obtain

\[
\ddot{\rho}_k = - \sum_i (\mathbf{k} \cdot \mathbf{v}_i)^2 e^{-ik \cdot x_i} \cdot \sum_{i, j, k \neq 0} [4 \pi e^2 i/m_e \gamma_i (1 + \beta_i \gamma_i^2)] e^{-ik \cdot x_j}.
\]

In regard of the velocity \( \mathbf{v}_i \) in the second term of the right-hand side of the above equation as the average value \( \mathbf{v} \) of the system, we have

\[
\ddot{\rho}_k = - \sum_i (\mathbf{k} \cdot \mathbf{v}_i)^2 e^{-ik \cdot x_i} \cdot \sum_{i, j, k \neq 0} [4 \pi e^2 i/m_e \gamma_i (1 + \beta_i \gamma_i^2)] e^{-ik \cdot x_j}.
\]

(10)

For sufficiently small \( k \), the first term can obviously be neglected in comparison with the second. Hence, we have

\[
\ddot{\rho}_k + [4 \pi e^2 i/m_e \gamma_i (1 + \beta_i \gamma_i^2)] \rho_k = 0,
\]

(11)

with

\[
\omega_p = [4 \pi e^2 i/m_e \gamma_i (1 + \beta_i \gamma_i^2)]^{1/2}.
\]

(12)

Thus, as a result of the Coulomb interaction, the relativistic electron density oscillates with the hot plasma frequency \( \omega_p \), just as the conventional theory used. From the definition (12) of \( \omega_p \), we can see that it contains relativistic modified factor \( \gamma(1 + \beta^2 \gamma^2) \). That is essentially different from the non-relativistic one. It is obvious that the increase in the velocity of the relativistic electron leads to a decrease in the plasma frequency \( \omega_p \).

One knows that both the collective oscillation and the random thermal motion are present simultaneously. Here we mainly care the collective effect of the Coulomb forces. As for a collective description, the rough criterion is

\[
[4 \pi e^2 i/m_e \gamma_i (1 + \beta_i \gamma_i^2)] \gg \langle (\mathbf{k} \cdot \mathbf{v}_i)^2 \rangle_{AV}.
\]

(13)

It means that for most particles, the collective term arising from the Coulomb forces in Eq. (10) is much greater than the individual term arising from the random thermal motion. Therefore, we note that the
strong force of interaction and high particle density favour collective behaviour, while high random thermal velocities oppose it.

For an isotropic velocity distribution, Eq. (13) becomes

\[ k^2 \ll \frac{12\pi n e^2}{m_c \gamma (1 + \beta^2 \gamma^2) \langle v_i^2 \rangle_{AV}} = \lambda_D^{-2}, \]  

(14)

where \( \lambda_D \) may be called the “Debye length”

\[ \lambda_D^2 = \frac{1}{3} \left[ \langle v_i^2 \rangle_{AV} \right] / \omega_p^2. \]  

(15)

Obviously, increasing the Lorentz factor \( \gamma \), as well as decreasing the plasma frequency \( \omega_p \), leads to a growth in the Debye length. From Eqs. (10) and (14), one can see that the organized behaviour is much more important in phenomena with distances greater than \( \lambda_D \), while the individual behaviour is favourable with distances shorter than \( \lambda_D \).

In conclusion, we have obtained the frequency of the hot plasma and the so-called Debye length with relativistic modification by studying the Coulomb interaction between relativistic electrons in hot plasma. To avoid complication in calculation and just expect to obtain a semi-quantitative result, we have roughly omitted the third term on the right-hand side of Eq. (3) and taken some approximation for the Coulomb interaction. We shall come back to reconsider these terms in details elsewhere. In this case, increasing the Lorentz factor \( \gamma \), as well as increasing the velocity of the electrons, leads to a decrease of the plasma frequency \( \omega_p \) and an increase of the Debye length. The lower plasma frequency is much easier for the relativistic electrons to behave collectively.

Finally, we only pay attention to the collective oscillation of relativistic electrons in the hot plasma. Actually, the organized behaviour and the random thermal motion are present simultaneously. We will discuss the later case in details elsewhere.

References


