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Constraint equation and loop expansion in light-cone ϕ^4 field theory

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Abstract. The constraint equation of the light-cone ϕ^4 field theory in 1 + 1 dimensions is the focus of the investigation of the non-triviality of the light-cone vacuum. The path-integral formalism developed from the Dirac–Bergmann algorithm is employed to calculate the vacuum expectation value instead of the constraint equation. The loop expansion given by Jackiw is used to calculate the effective potential as a function of the zero-mode field ω up to two-loop order. The minimum of the effective potential is at the vacuum expectation value of the ϕ field. A criterion for the first- or second-order phase transition is given by the effective potential and the critical coupling constants are calculated. Under the assumption that the zero and non-zero modes are classical fields, the constraint equation is used to solve for ω expressed in terms of the non-zero modes in a power series in $\hbar$. A static soliton solution for the non-zero modes is obtained from the constraint equation.

1. Introduction

For many years, the light-cone vacuum was considered to be simple. It carries no longitudinal momentum ($k^+ = 0$), while the momentum operators for fermions have $k^+ > 0$ and the vacuum state of the free Hamiltonian is an eigenstate of the light-cone Hamiltonian with interactions in many theories. A Fock-space basis constructed from the physical vacuum has been applied to the Hamiltonian of quantum chromodynamics to produce hadron spectra in the discretized light-cone quantization method [1]. In spite of the fact that the zero modes occupy only a very small portion of the light-cone theory, their importance has been emphasized in vacuum-diagram calculations [2] and in the null-plane quantization of scalar fields [3]. Recent studies commonly indicate significant and indispensable demands for the zero modes in various field theories [4–16]. The inclusion of zero modes can remedy shortcomings in the discretized light-cone quantization method and reduce the contradiction between the simple light-cone vacuum and the usual expectations for the structure of the QCD vacuum [17]. It is concluded, for the non-trivial behaviour and effects of the light-cone vacuum, that the bosonic zero modes remove certain non-covariant and quadratically divergent terms in the fermion self energy in the discretized light-cone quantization of scalar-coupled Yukawa theory [7], the zero modes enter the internal lines at any order of Feynman diagrams for the ϕ^3 field theory [12], θ-vacua exist due to a topological large-gauge transformation and exhibit non-vanishing fermion condensates [5, 9] and the zero mode has a large effect on the spectra of Hamiltonian and field operators [10].

The non-triviality of the light-cone vacuum is more apparently exposed in the pure glue theory in two space–time dimensions, since the zero mode has one dynamical independent field component which is beyond control [14]. The light-cone gauge $A^+ = 0$ cannot be reached since the zero mode in A^+ is gauge invariant [11, 14] and the other allowable gauge has a Gribov ambiguity. In contrast to this complexity, the ϕ^4 field theory in 1+1 dimensions is a relatively simple sample for studying non-trivial vacuum properties since the zero mode is in a constraint equation. This non-triviality is exhibited through the development of non-zero vacuum expectation values of the ϕ field when the coupling constant is smaller than a critical value. Since the potential in the ϕ^4 theory has two minima where the vacuum states are defined, the spontaneous symmetry-breaking phenomenon exists not only in the conventional equal-time field theory but also in the light-cone field theory. The zero mode in the vacuum undertakes the phase transition between the symmetric and broken phases.

The Dirac–Bergmann algorithm [18] is applied to obtain a constraint equation (see equation (3c)) for the zero mode of the real scalar field [3–5, 10]. This equation is also most conveniently obtained by integrating the equation of motion for the ϕ field [19]. The constraint equation is investigated with regard to the spontaneous (reflection) symmetry breaking [5, 10, 19, 20]. The vacuum expectation value $\langle\phi\rangle$ was obtained in the tree approximation plus a lowest-order correction in [5, 19]. The constraint equation tells us that the zero-mode field is not dynamically free but a functional of the non-zero-mode field. Lying in this observation, the zero-mode field operator can be expressed in terms of the creation and annihilation operators for the non-zero modes [20] or the projection operators for the particle Fock states [10]. In [20] two ansätze for the zero mode ω were made and an effective potential turned out to be a linear combination of $\langle\phi\rangle^2$ and $\langle\phi\rangle^4$ terms. A critical coupling constant $\lambda_c = 40.0\ \mathrm{m}^2$ was obtained when the curvature of the potential at $\langle\phi\rangle = 0$ changes its sign. In [10], the zero mode is diagonally constructed from the Fock states with Tamm–Dancoff truncation. The vacuum expectation value as a function of the coupling constant is obtained by solving the constraint equation by the numerical method or by δ expansion. The critical coupling constant is reported to be $\lambda_c = 59.5\ \mathrm{m}^2$ by searching for a convergent solution of the constraint equation. When the zero and non-zero modes are assumed to be classical fields, the ω can be solved exactly from the constraint equation. While the zero mode is expressed as a power series in the coupling constant in [19], we obtain the ω as a power series in the Planck constant $\hbar$ (see section 4).

For the ϕ^4 theory, the ϕ field is decomposed into the zero and non-zero modes and the two types of mode couple to each other by a cubic interaction term. Via the constraint θ_3 the non-zero modes influence the zero modes not only in tree-level interaction but also in higher-order diagrams, i.e. loop graphs. The higher-order interactions create a high non-linear vacuum structure which causes some long-range and non-linear phenomena such as the spontaneous symmetry breaking, bound states and resonances. Investigations of these phenomena thus give an insight into the non-triviality of the vacuum. In order to study the non-linear phenomena, a loop expansion was constructed by Jackiw [21] to calculate richer non-linear structure in the effective potential which represents the complicated vacuum interactions. The effective potential is defined by the effective action, which is formulated in a WKB loop expansion series. Before performing loop expansion, the Lagrangian is shifted by a constant field (in the static case). The propagator contains the constant field in its denominator and the coupling constant in the cubic interaction depends linearly on the constant field. When the constant field is explained as the vacuum expectation value of the quantum field, the loop expansion yields non-perturbative results representing some vacuum effects [22].

The effective potential up to two loops contains logarithmic functions of the zero-mode field and thus exhibits richer non-linear structure than the potential in [20] which has only quadratic and quartic terms as mentioned above. In the first paper of [10] the constraint equation is decomposed into a chain of equations which are solved numerically to obtain the vacuum expectation value. This procedure amounts to the use of the effective potential of a higher-order polynomial of the vacuum expectation value. Here we indicate that another type of loop expansion has recently been performed for the ϕ field in [23] and an effective potential has been calculated at one-loop order using the non-perturbative techniques of light-cone quantization for the Hamiltonian formalism [1].

In $3+1$ dimensions, the zero modes depend on the other two coordinates x^1 and x^2. This dependence increases the difficulty in evaluating vacuum graphs. Thus for simplicity, the ϕ^4 field theory is studied here in $1+1$ dimensions with a single spin-0 field ϕ. The paper is organized as follows. In the next section we prove that the effective action can be applied to calculate the vacuum expectation value instead of the constraint equation. Then, the loop-expansion formulae given by Jackiw [21] are employed to calculate the effective potential up to the two-loop order. In section 3, this effective potential is used to give the criterion for the first- or second-order phase transition and two equations for the evaluation of critical coupling constants. In section 4, while the zero and non-zero modes are taken as classical fields, the zero modes are solved from the constraint equation and expanded in power series in $\hbar$. Inversely, this gives an insight into the non-zero modes when the solutions are compared with the vacuum expectation values obtained from the effective potential. A restriction on the renormalized mass and coupling constant is induced. In section 5, a non-topological soliton solution for the the non-zero-mode field is obtained from the constraint equation. Section 6 presents numerical calculations on the effective potential, vacuum expectation value and critical coupling constants together with a discussion. The final section contains conclusions.

2. Effective potential

The light-front form is obtained from the instant form in the infinite-momentum frame, while light-cone variables are linear combinations of corresponding instant-form variables [24]. The Poincaré group realized on the null plane $(t+z=0)$ contains a maximum number of kinematic generators and a minimum number of dynamical generators [25]. The Hamiltonian constructed in the null-plane quantization is the evolution operator of the light-cone time. The Lorentz boost operator along the longitudinal direction is interaction free so that the theory is Lorentz invariant. However, the interaction-dependent rotation operator leads to the theory with only approximate rotational invariance.

The light-cone time and space coordinates are defined as $x^+ = t+z$ and $x^- = t-z$ and the light-cone momenta are $k^+ = k^0 + k^3$ and $k^- = k^0 - k^3$. This leads to the dot product of two light-cone variables in $1+1$ dimensions: $a \cdot b = \frac{1}{2}(a^+b^- + a^-b^+)$. In the Fourier transform of the field $\varphi(x)$ in the x^- coordinate,

$$\varphi(x) = \int \frac{\mathrm{d}k^+}{2\pi} \exp\left(-\mathrm{i}\frac{k^+x^-}{2}\right) \varphi(k^+, x^+)$$

the zero-mode field is just $\varphi(k^+ = 0, x^+)$ independent of the x^-. With the following notation [26]

$$\partial_+ = \frac{\partial}{\partial x^+} \qquad \partial_- = \frac{\partial}{\partial x^-} \qquad \partial^+ = 2\partial_- \qquad \partial^- = 2\partial_+$$

the Lagrangian of the light-cone ϕ^4 field theory in $1+1$ dimensions is written as

$$\mathcal{L}(\phi) = \frac{1}{2}\partial^+\phi\partial^-\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{1}$$

where m is the mass and λ is the coupling constant. The field ϕ is decomposed into the zero modes ω and non-zero modes φ: $\varphi = \phi - \omega$, where

$$\omega = \frac{1}{2L}\int_{-L}^{+L} \mathrm{d}x^- \, \phi(x)$$

and L is the boundary of a spatial box in which the field is quantized. In terms of ω and φ, the Lagrangian is written as

$$\mathcal{L}(\varphi, \omega) = \frac{1}{2}\partial^+\varphi\partial^-\varphi - \frac{1}{2}m^2(\varphi+\omega)^2 - \frac{\lambda}{4!}(\varphi+\omega)^4. \tag{2}$$

Since the light-cone Lagrangian is maximally singular, two primary constraints are obtained from the definition of the canonical momenta with respect to φ and ω (denoted by π_φ and π_ω). Applying the Dirac–Bergmann algorithm, three constraints of second class

$$\theta_1 = \pi_\varphi(x) - \partial^+\varphi(x) \approx 0 \tag{3a}$$

$$\theta_2 = \pi_\omega \approx 0 \tag{3b}$$

$$\theta_3 = m^2\omega + \frac{\lambda}{3!}\omega^3 + \frac{1}{2L}\int_{-L}^{+L} \mathrm{d}x^- \, \frac{\lambda}{3!}\left[\varphi^3(x) + 3\varphi^2(x)\omega\right] \approx 0 \tag{3c}$$

and their Dirac brackets have been obtained in [5, 20]. Alternatively, a symplectic method [27] was applied to obtain the same Dirac brackets without the classification of primary and secondary constraints as well as the first- and second-class constraints [28].

Before proceeding with the path-integral formalism, the limit as $L \to \infty$ was taken [5]. In this limit, the definition of ω gives the L-independent zero-mode field. For the ϕ field in equilibrium, the zero-mode field ω is a constant field which is independent of space–time. Notice, however, that ω is functionally dependent on the φ field through the constraint θ_3 and that both ω and φ are operators [10, 20] which include quantum fluctuations. Nevertheless, in the path-integral formalism, ω and φ are taken as classical fields. On the basis of the Dirac brackets, we write down the vacuum persistence amplitude in the absence of the external source according to the formalism of path-integral quantization of field theories with second-class constraints [29],

$$Z(0) = |2\,\mathrm{Det}\,(\partial_x^+\delta(x^- - y^-))|^{1/2} \int \mathcal{D}\varphi \exp\left[\frac{\mathrm{i}}{\hbar}\int \mathrm{d}^2x\, \mathcal{L}(\varphi, \omega)\right]. \tag{4}$$

The determinant preceding the functional integral is simply a constant, and not important in calculations of conventional Feynman diagrams. In later calculations it is neglected. The connected generating functional is

$$W(0) = -\mathrm{i}\hbar \ln Z(0) \tag{5}$$

and the effective action $\Gamma(\omega)$ is just $W(0)$. Taking the derivative of $\Gamma(\omega)$ with respective to ω gives

$$\frac{\mathrm{d}\Gamma(\omega)}{\mathrm{d}\omega} = -\frac{1}{Z[0]}\int \mathcal{D}\varphi \exp\left[-\frac{\mathrm{i}}{\hbar}\int \mathrm{d}x^+\, H_\mathrm{c}\right]\int \mathrm{d}x^+\, 2L\theta_3 \tag{6}$$

where the canonical Hamiltonian is

$$H_\mathrm{c} = \int_{-L}^{+L} \mathrm{d}x^- \left[\frac{1}{2}m^2(\varphi+\omega)^2 + \frac{\lambda}{4!}(\varphi+\omega)^4\right]. \tag{7}$$

It is obvious that $d\Gamma(\omega)/d\omega = 0$ is equivalent to $\theta_3 = 0$. The estimate of the vacuum expectation value $\langle\phi\rangle$ from the constraint $\theta_3 = 0$ is translated into a calculation of $d\Gamma(\omega)/d\omega = 0$. For the evaluation of the effective action $\Gamma(\omega)$ from the path-integral formalism, the frame-independent loop-expansion formalism given by Jackiw [21] is adopted in this paper. In [21], an expansion in powers of $\hbar$ was made for the effective action $\Gamma(\hat{\phi})$ after shifting the field ϕ by a constant $\hat{\phi}$. In this spirit, (2) suggests a 'shift' of the field φ by a constant field ω. Thus, in any calculation with the formulae given by Jackiw, one must bear in mind that the point $k^+ = 0$ in the integration over k^+ must be omitted.

In [12], the author first solved the secondary constraint equation for the zero and non-zero modes to express ω in terms of φ, then expanded it in a power series in the coupling constant and finally obtained an expansion about the coupling constant for the interaction Hamiltonian of the field φ. In such an expansion the author recognized that zero modes propagate along the internal lines in any order of the Feynman diagrams for the field φ. In contrast to this procedure, we first expand the effective action in a power series in $\hbar$, then invoke the constraint $\theta_3 = 0$ in this expansion series, or equivalently, implement $d\Gamma(\omega)/d\omega = 0$. In other words, according to equations (3.7) to (3.11) of the second paper of [5], we first expand $Z(0)$ before integrating ω, then carry out the integration over ω and thus implement the constraint $\theta_3 = 0$.

In the case of an external source $J(x)$, a term $J(x)\varphi(x)$ is added to the Lagrangian in (4) to obtain the vacuum persistence amplitude $Z(J)$. In terms of $Z(J)$, the connected generating functional $W(J)$ is defined by

$$W(J) = -i\hbar \ln Z(J). \tag{8}$$

The effective action is obtained from $W(J)$ by a Legendre transformation

$$\Gamma(\bar{\varphi}) = W(J) - \int d^2x\, \bar{\varphi}(x) J(x) \tag{9}$$

with $\bar{\varphi}(x) = \delta W(J)/\delta J(x)$. The effective potential $V(\omega)$ is defined from the effective action by setting $\bar{\varphi}(x)$ to be a constant field ω, which is reached by 'shifting' $\varphi(x)$ by the ω in the Lagrangian, and extracting an overall factor of $1+1$ dimension volume,

$$\Gamma(\omega) = -V(\omega) \int d^2x. \tag{10}$$

The constraint $d\Gamma(\omega)/d\omega = 0$, i.e. $dV(\omega)/d\omega = 0$, produces exactly the vacuum expectation value of the field ϕ. By expanding $W(J)$ and $\delta W(J)/\delta J$ as a power series in $\hbar$, the effective potential $V(\omega)$ in loop expansion is given by [21]

$$V(\omega) = V_{\text{tree}}(\omega) - \frac{1}{2} i\hbar \int [d^2k]' \ln\det iD^{-1}(\omega; k) + i\hbar \left\langle \exp\left[\frac{i}{\hbar}\int d^2x\, \mathcal{L}_I(\varphi, \omega)\right]\right\rangle \tag{11}$$

where $[d^2k]'$ is the measure of the light-cone momentum in $1+1$ dimensions without the point $k^+ = 0$ since we perform the loop expansion for the field φ. The first term $V_{\text{tree}}(\omega)$ is the classical potential (tree approximation). The second term is the one-loop effective potential which involves a logarithm and the third term expresses the effective potential generated from multi-loop diagrams. $\mathcal{L}_I(\varphi, \omega)$ is composed of cubic and quartic terms in $\varphi(x)$. *$D(\omega; k)$ is the propagator in momentum space and its explicit dependence on ω* results from the field φ 'shifted' by ω. The practical calculation as outlined above starts from the Lagrangian for the spin-0 ϕ^4 field,

$$\mathcal{L}(\phi) = \frac{1}{2}\partial^+\phi\partial^-\phi - \frac{1}{2}(m_0^2 + \delta m^2)\phi^2 - \frac{\lambda_0 + \delta\lambda}{4!}\phi^4 \tag{12}$$

where m_0 and λ_0 are the finite, but undetermined, mass and coupling constant, respectively. The counterterms δm^2 and $\delta\lambda$ are given in the form of power series in $\hbar$. We 'shift' the field φ by a constant field ω. The 'shifted' Lagrangian is

$$\mathcal{L}(\varphi,\omega)=\frac{1}{2}\partial^+\varphi\partial^-\varphi-\frac{1}{2}\mu^2\varphi^2-\frac{\lambda_0+\delta\lambda}{6}\omega\varphi^3-\frac{\lambda_0+\delta\lambda}{4!}\varphi^4 \tag{13}$$

with

$$\mu^2=m_0^2+\delta m^2+\frac{\lambda_0+\delta\lambda}{2}\omega^2. \tag{14}$$

In the shifted Lagrangian the induced mass μ depends on ω and an ω-dependent cubic interaction is obtained. The ω-dependent propagator in the momentum space is defined by the new quadratic term,

$$D(\omega;p)=\frac{\mathrm{i}}{p^+p^--\mu^2+\mathrm{i}\epsilon} \tag{15}$$

and the classical potential is

$$V_{\mathrm{tree}}(\omega)=\frac{m_0^2+\delta m^2}{2}\omega^2+\frac{\lambda_0+\delta\lambda}{4!}\omega^4. \tag{16}$$

For a single field ϕ, the propagator $D(\omega,p)$ is diagonal in momentum space and the determinant is thus removed. The one-loop effective potential corresponding to figure 1(a) is written as

$$V_1(\omega)=-\frac{\mathrm{i}\hbar}{2}\int[\mathrm{d}^2k]'\ln\mathrm{i}D^{-1}(\omega;k)=-\frac{\hbar}{8\pi}[(2\Lambda^2+\mu^2)\ln(2\Lambda^2+\mu^2)-\mu^2\ln\mu^2] \tag{17}$$

where Λ is a cut-off of the high momenta k^+ and k^-. The two-loop effective potential is

$$V_2(\omega)=\frac{\hbar^2\lambda_0}{8}\int[\mathrm{d}^2k]'[\mathrm{d}^2l]'D(\omega;k)D(\omega;l)$$
$$-\frac{\mathrm{i}\hbar^2\lambda_0^2}{12}\omega^2\int[\mathrm{d}^2k]'[\mathrm{d}^2l]'\,D(\omega;k+l)D(\omega;k)D(\omega;l) \tag{18}$$

where the first term corresponds to figure 1(b) and the second term to figure 1(c). We obtain

$$V_2(\omega)=\frac{\hbar^2\lambda_0}{128\pi^2}\ln^2\left(1+\frac{2\Lambda^2}{\mu^2}\right)+\frac{\hbar^2\lambda_0^2\omega^2}{96\pi^2}\left\{\left(\frac{1}{4\Lambda^2}-\frac{\pi}{\mu^2\sqrt{3+(8\Lambda^2/\mu^2)}}\right)\right.$$
$$\left.\left[\ln\left(1+\frac{2\Lambda^2}{\mu^2}\right)+\ln\left(1+\frac{\mu^2}{2\Lambda^2}\right)\right]+\frac{2\pi}{3\sqrt{3}\mu^2}\ln\left(1+\frac{\mu^2}{2\Lambda^2}\right)\right\}. \tag{19}$$

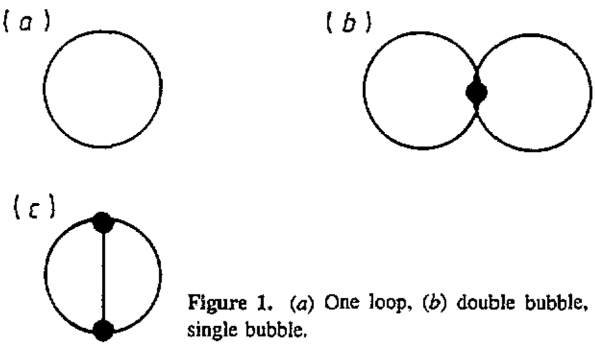

Figure 1. (a) One loop, (b) double bubble, (c) 'radiatively' corrected single bubble.

Expanding the counterterms δm^2 and $\delta\lambda$ in powers of $\hbar$ gives

$$\delta m^2 = \hbar\delta m_1^2 + \hbar^2\delta m_2^2 + \ldots \qquad \delta\lambda = \hbar\delta\lambda_1 + \hbar^2\delta\lambda_2 + \ldots . \tag{20}$$

The values of these counterterms are obtained by cancelling divergent terms appearing in $V_1(\omega)$ and $V_2(\omega)$ which are expanded to the order of $\hbar^2$. We find

$$\delta m_1^2 = \frac{\lambda_0}{8\pi}\ln(2e\Lambda^2) + \delta\bar{m}_1^2 \tag{21}$$

where $\delta\bar{m}_1^2$ is a finite, but arbitrary, quantity. This result resembles other low-dimensional calculations, for example, in [30]. Since no divergent terms need to be cancelled by δm_2^2, $\delta\lambda_1$ and $\delta\lambda_2$, we leave these finite, but arbitrary. They might be determined by certain renormalization conditions [31] relating to the spin-0 meson mass and empirical coupling constant. In terms of these quantities, the finite effective potential up to the two-loop order is

$$\begin{aligned} V(\omega) &= V_{\text{tree}}(\omega) + V_1(\omega) + V_2(\omega) \\ &= \frac{1}{2}m_R^2\omega^2 + \frac{1}{4!}\lambda_R\omega^4 + \frac{\hbar}{8\pi}a\ln a + \frac{\hbar^2\lambda_0}{128\pi^2}\ln^2 a + \frac{\hbar^2\lambda_0}{64\pi^2}\ln a \\ &\quad + \frac{\hbar^2}{8\pi}(\delta\bar{m}_1^2 + \frac{\delta\lambda_1}{2}\omega^2)\ln a \end{aligned} \tag{22}$$

with $a = m_0^2 + (\lambda_0/2)\omega^2$, renormalized mass square $m_R^2 = m_0^2 + \hbar\delta\bar{m}_1^2 + \hbar^2\delta m_2^2$ and renormalized coupling constant $\lambda_R = \lambda_0 + \hbar\delta\lambda_1 + \hbar^2\delta\lambda_2$. The non-linear logarithmic structure due to the loop graphs is explicitly exhibited.

3. Criterion for first- or second-order phase transition

When $m_0^2 < 0$, $\ln a$ is imaginary within the region $-\sqrt{-(2/\lambda_0)m_0^2} \leqslant \omega \leqslant \sqrt{-(2/\lambda_0)m_0^2}$. When $m_0^2 = 0$, $V(\omega)$ tends to $+\infty$ as $\omega \to 0$ since $\ln^2 a \gg |\ln a|$. Hence the inequality $m_0^2 \leqslant 0$ is not proper in the description of spontaneous symmetry breaking. The condition $m_0^2 > 0$ ensures that the effective potential is real and finite if $\omega \neq \pm\infty$. To illustrate the choice of the quantities in m_R^2 and λ_R, we consider a single pion field. Without loss of generality, we may assume that $m_0^2 = \gamma m_\pi^2$, $\delta\bar{m}_1^2 = -m_0^2 - m_\pi^2$ and $\delta m_2^2 = 0$, leading to $m_R^2 = -m_\pi^2$, and $\lambda_0 = \alpha m_\pi^2$, $\delta\lambda_1 = \beta m_\pi^2$ and $\delta\lambda_2 = 0$, leading to $\lambda_R = (\alpha+\beta)m_\pi^2$. Here, m_π is the observed pion mass. Positive values of m_0^2 and λ_R require that $\gamma > 0$ and $\alpha + \beta > 0$.

We have obtained the effective potential. Yet it is not known whether it is able to describe phase transitions of first or second order. Below, we give a criterion which states that the effective potential causes a phase transition of a first- or second-order nature. In practical calculations, $\hbar$ is set to 1. The derivative of $V(\omega)$ with respect to ω is immediately obtained from (22),

$$\begin{aligned} \frac{dV(\omega)}{d\omega} &= m_\pi^2\omega\left\{\frac{\alpha}{8\pi} - 1 + \frac{\alpha+\beta}{6}\omega^2 + \left(\frac{\alpha}{64\pi^2} - \frac{\gamma+1}{8\pi} + \frac{\beta}{16\pi}\omega^2\right)\frac{\alpha m_\pi^2}{a}\right. \\ &\quad \left. + \left(\frac{\alpha+\beta}{8\pi} + \frac{\alpha^2 m_\pi^2}{64\pi^2 a}\right)\ln a\right\}. \end{aligned} \tag{23}$$

Let $dV(\omega)/d\omega = 0$. Except for $\omega = 0$, the other two solutions satisfy $(m_\pi^2\omega)^{-1}dV(\omega)/d\omega = 0$. If the phase transition is of second order, the two solutions must also satisfy $\omega = 0$ at the critical point. In this case, α, β and γ must satisfy

$$-1+\left(\frac{\alpha^2}{64\pi^2}-\frac{\alpha}{8\pi}\right)\frac{1}{\gamma}+\frac{\alpha+\beta}{8\pi}\ln m_0^2+\frac{\alpha^2}{64\pi^2}\frac{\ln m_0^2}{\gamma}=0. \tag{24}$$

If α, β and γ do not satisfy (24), then the vacuum expectation value of the ϕ field at the critical point is not equal to zero. This situation corresponds to a second-order phase transition. Equation (24) is thus a criterion which states that the theory describes the first- or second-order phase transition depending on the choice of the quantities in m_R^2 and λ_R. If, initially, we choose values for α and γ, then β can be obtained easily from (24) and the critical coupling constant for the second-order phase transition is

$$\lambda_c''=-\frac{\alpha^2 m_\pi^2}{8\pi\gamma}+\frac{m_\pi^2}{\ln m_0^2}\left(8\pi+\frac{\alpha}{\gamma}-\frac{\alpha^2}{8\pi\gamma}\right). \tag{25}$$

It has been shown in conventional equal-time field theory in 3 + 1 dimensions that the effective potential is able to describe a first-order phase transition [32]. This can be done in the light-cone theory provided the vacuum expectation value at the critical point is away from the zero $\langle\phi\rangle=\phi_c\neq 0$. The effective potential and its derivative (23) are zero when $\omega=\phi_c$. The formula $V(\phi_c)=0$ gives

$$\alpha+\beta=\left(\frac{\phi_c^4}{12}+\frac{\phi_c^2}{8\pi}\ln a\right)^{-1}\left[\phi_c^2+\frac{1}{4\pi}\left(1-\frac{\alpha}{8\pi}\right)\ln a-\frac{\alpha}{64\pi^2}\ln^2 a\right] \tag{26}$$

which is inserted into $(m_\pi^2\phi_c)^{-1}\mathrm{d}V(\phi_c)/\mathrm{d}\phi_c=0$ to give

$$\left[\frac{\phi_c^4}{192\pi^2}(1+\ln a)+\frac{\phi_c^2\ln^2 a}{256\pi^3}\right]\alpha^2+\left[\frac{5\phi_c^4}{24\pi}-\frac{\eta\phi_c^2\ln a}{48\pi^2}-\frac{\eta\ln^2 a}{64\pi^3}-\frac{\eta\phi_c^2\ln^2 a}{96\pi^2}-\frac{\eta\ln^3 a}{128\pi^2}\right]\alpha$$
$$+\frac{\eta\phi_c^4}{3}+\frac{\eta\phi_c^2\ln a}{6\pi}+\frac{\eta\ln^2 a}{8\pi^2}=0 \tag{27}$$

with $\eta=a/m_\pi^2$. If the vacuum expectation value at the critical point, ϕ_c, is assumed to be measured in experiments, then a is the only input to calculate the α by (27), $\gamma=\eta-(\alpha\phi_c^2/2)$ and the critical coupling constant at the first-order phase transition $\lambda_c'=(\alpha+\beta)m_\pi^2$ by (26).

4. Expansion of the constraint equation in powers of $\hbar$

The constraint equation is central to the light-cone ϕ^4 field theory in the two space–time dimensions. In section 2 we performed the loop expansion by employing the vacuum persistence amplitude formalism. But the effective potential can only provide the vacuum expectation value and not the constraint equation. Hence, we return to the constraint equation to learn more about the field, assuming that φ and ω are classical fields. The constraint equation is now a cubic equation in ω which is easily solved without the need to order the field operators. Let

$$b=\frac{1}{2L}\int_{-L}^{+L}\mathrm{d}x^-\,\varphi^2(x^-) \tag{28}$$

$$c=\frac{1}{2L}\int_{-L}^{+L}\mathrm{d}x^-\,\varphi^3(x^-). \tag{29}$$

The constraint equation is written as

$$\omega^3+\lambda\left(\frac{6m_R^2}{\lambda_R}+3b\right)\omega+c=0. \tag{30}$$

If we rescale the field: $\varphi \to (\hbar)^{1/2}\varphi$, b and c are of order $\hbar$ and $\hbar^{3/2}$ respectively. Since fractional powers of $\hbar$ cannot occur, c begins with order $\hbar^2$. We expand b and c in a series of powers of $\hbar$,

$$b = b_1\hbar + b_2\hbar^2 + \ldots \qquad c = c_2\hbar^2 + c_3\hbar^3 + \ldots. \tag{31}$$

The three solutions of (30) are

$$\omega_1 = A + B \tag{32a}$$

$$\omega_2 = -\frac{1}{2}(A+B) + \frac{\sqrt{3}}{2}\mathrm{i}(A-B) \tag{32b}$$

$$\omega_3 = -\frac{1}{2}(A+B) - \frac{\sqrt{3}}{2}\mathrm{i}(A-B) \tag{32c}$$

with

$$\begin{aligned} A &= \left\{-\frac{c}{2} + \left[\frac{c^2}{4} + \left(\frac{2m_R^2}{\lambda_R} + b\right)^3\right]^{1/2}\right\}^{1/3} \\ B &= \left\{-\frac{c}{2} - \left[\frac{c^2}{4} + \left(\frac{2m_R^2}{\lambda_R} + b\right)^3\right]^{1/2}\right\}^{1/3} \end{aligned} \tag{33}$$

Substituting (20) and (31) into (32) and (33) and expanding in power series in $\hbar$, we obtain

$$\omega_1 = -\frac{\lambda_0 c_2}{6m_0^2}\hbar^2 + \ldots \tag{34a}$$

$$\omega_2 = -\bar{\omega} - \frac{d_1}{6}\bar{\omega}\hbar + \left(-\frac{c_2}{2\bar{\omega}^2} - \frac{d_2}{6}\bar{\omega} + \frac{5d_1^2}{72}\bar{\omega}\right)\hbar^2 + \ldots \tag{34b}$$

$$\omega_3 = \bar{\omega} + \frac{d_1}{6}\bar{\omega}\hbar + \left(-\frac{c_2}{2\bar{\omega}^2} + \frac{d_2}{6}\bar{\omega} - \frac{5d_1^2}{72}\bar{\omega}\right)\hbar^2 + \ldots \tag{34c}$$

with

$$\bar{\omega} = \left(-\frac{6m_0^2}{\lambda_0}\right)^{1/2} \qquad d_1 = \frac{3\lambda_0}{2m_0^2}\left(\frac{2\delta m_1^2}{\lambda_0} - \frac{2m_0^2}{\lambda_0^2}\delta\lambda_1 + b_1\right)$$

$$d_2 = \frac{d_1^2}{3} + \frac{3\lambda_0}{2m_0^2}\left(\frac{2\delta m_2^2}{\lambda_0} - \frac{2m_0^2}{\lambda_0^2}\delta\lambda_2 - \frac{2\delta\lambda_1\delta m_1^2}{\lambda_0^2} + \frac{2m_0^2}{\lambda_0^3}\delta\lambda_1^2 + b_2\right).$$

Usually, b and c are unknown. However, we can obtain their numerical values by comparing (34) with the maximum and two minima of the effective potential. Here, we must ensure that if ω_1 corresponds to the maximum, ω_2 and ω_3 correspond to the two minima. From the canonical Hamiltonian (7), we evaluate the energy difference between any two of ω_1, ω_2 and ω_3:

$$\left.\frac{H_c}{2L}\right|_{\omega=\omega_i} - \left.\frac{H_c}{2L}\right|_{\omega=\omega_j} = -(\omega_i^2 - \omega_j^2)\frac{\lambda_R}{8}\left[2\left(\frac{2m_R^2}{\lambda_R} + b\right) + \omega_i^2 + \omega_j^2\right]. \tag{35}$$

It is hard to judge which of the energies corresponding to ω_i and ω_j is the higher by direct substitution of (32) or (34) into (35). Alternatively, an approximation can be made to reach the destination. Since $\int_{-L}^{+L} \mathrm{d}x^-\,\varphi(x^-) = 0$, $\varphi(x^-)$ may be an antisymmetric function. We may assume that $\varphi(x^-)$ is a function which changes smoothly and modestly with respect to the variable x^-. In general, we can always decompose $\varphi(x^-)$ into a positive part $\varphi_+(x^-)$ and a negative part $\varphi_-(x^-)$

$$\begin{aligned}&\varphi(x^-)=\varphi_+(x^-)+\varphi_-(x^-) \qquad \varphi_+(x^-)=f_+(x^-)+\delta f_+(x^-)\\&\varphi_-(x^-)=-f_-(x^-)-\delta f_-(x^-)\end{aligned} \tag{36}$$

with $f_\pm(x^-)\geqslant 0$, $f_\pm(x^-)\gg|\delta f_\pm(x^-)|$, $f_\pm(x^-)=f_\mp(-x^-)$ and $\delta f_\pm(x^-)\neq\delta f_\mp(-x^-)$, which guarantees that $\varphi(x^-)$ is a quite rough antisymmetric function of x^-. Simple derivation shows that b is of the order $(1/2L)\int_{-L}^{+L}\mathrm{d}x^-\, f_\pm^2(x^-)$ and c is of the order $(1/2L)\int_{-L}^{+L}\mathrm{d}x^-\, f_\pm^2(x^-)\delta f_\pm(x^-)$. Thus, neglecting c in the constraint equation we find that the three solutions in (32) are

$$\omega_1=0 \qquad \omega_2=\sqrt{3}\mathrm{i}\left(\frac{2m_\mathrm{R}^2}{\lambda_\mathrm{R}}+b\right)^{1/2} \qquad \omega_3=-\sqrt{3}\mathrm{i}\left(\frac{2m_\mathrm{R}^2}{\lambda_\mathrm{R}}+b\right)^{1/2}. \tag{37}$$

This gives $H_\mathrm{c}|_{\omega=\omega_1}\geqslant H_\mathrm{c}|_{\omega=\omega_2}=H_\mathrm{c}|_{\omega=\omega_3}$ where the inequality holds in the broken phase and the equality at the critical point of phase transition.

In the vacuum state, ω_1, ω_2 and ω_3 coincide with the three solutions given by (23). The ω_1 in (34) corresponds to the maximum and ω_2 and ω_3 to the minima in the effective potential. Inserting $\omega_1=0$ into (30) gives $c=0$. Subsequently, $\omega_2=-\omega_3$ gives

$$b=-\frac{\omega^2}{3}-\frac{2m_\mathrm{R}^2}{\lambda_\mathrm{R}}.$$

Let $\omega=1.21$, $m_\mathrm{R}^2=-m_\pi^2$ and $\lambda_\mathrm{R}=4m_\pi^2$ (see section 6), then $b=0.012$. In the tree approximation the non-vanishing vacuum expectation value stems solely from the self-interaction of zero modes as seen from (34). The influence of non-zero modes starts from the order of $\hbar$. In an arbitrary state, c may not be zero. Then $\omega_1\neq 0$ and $\omega_2\neq-\omega_3$ begin from the order of $\hbar^2$. In such a situation, $\varphi(x^-)$ is not an antisymmetric function of x^-. If $\varphi(x^-)$ is an antisymmetric function of x^-, then $c=0$. The solutions of the constraint equation with $c=0$ have the property $\omega_1=0$ and $\omega_2=-\omega_3$. This leads to the conclusion that the classical non-zero-mode field can be described by an antisymmetric function if the effective potential is a symmetric function of the zero-mode field ω. In the absence of external sources such an effective potential exists, for example, in the $1+1$ Ising model [32] and is explicitly shown by the $V(\omega)$ in (22). When c is not zero but small, $\varphi(x^-)$ is slightly different from the antisymmetric function.

For a real scalar ϕ field, the zero modes ω and non-zero modes $\varphi(x)$ projected from ϕ are real. To keep the three ω real in (32), it follows that $A=C+\mathrm{i}D$ and $B=C-\mathrm{i}D$. Equation (32) is rewritten as

$$\omega_1=2C \qquad \omega_2=-C-\sqrt{3}D \qquad \omega_3=-C+\sqrt{3}D. \tag{38}$$

Comparison of (38) with (34) gives

$$C=-\frac{\lambda_0 c_2}{12m_0^2}\hbar^2+\dots \qquad D=\frac{1}{\sqrt{3}}\left[\bar\omega+\frac{d_1}{6}\bar\omega\hbar+\left(\frac{d_2}{6}\bar\omega-\frac{5d_1^2}{72}\bar\omega\right)\hbar^2+\dots\right]. \tag{39}$$

b and c are real from their definitions and so is $m_\mathrm{R}^2/\lambda_\mathrm{R}$ by virtue of (30). D contributes to the three ω from the zeroth order of $\hbar$ and, thus, we can conclude that A and B are in general complex, which through (33) leads to

$$\frac{c^2}{4}+\left(\frac{2m_\mathrm{R}^2}{\lambda_\mathrm{R}}+b\right)^3<0. \tag{40}$$

Equation (40) for the parameters m_R^2 and λ_R is the restriction imposed by the real scalar field. If $c = 0$, (40) becomes $(2m_R^2/\lambda_R) + b < 0$ which guarantees that ω_2 and ω_3 are real in (37). In practical numerical calculations in section 6 this restriction is satisfied.

5. Classical soliton solution of the constraint equation

The non-zero value of b (and c) in the last section implies that the φ field is not localized and does not vanish at $x^- \to \pm\infty$ provided it is not singular. If the φ field is a single real plane wave, then $c = 0$. In analogy to the conventional equal-time field theory [33], we investigate the existence of static soliton solutions for the non-zero modes starting from the constraint equation. The soliton solution for the φ field is stable and localized in a finite region in space at all times and has a finite energy. If the φ field is localized, the b in (28) and c in (29) are zero. The constraint equation in this case is

$$\omega^3 + \frac{6m_R^2}{\lambda_R}\omega = 0 \tag{41}$$

where the renormalized mass and coupling constant are used. This gives three solutions in the tree approximation,

$$\omega_{1\mathrm{tree}} = 0 \qquad \omega_{2\mathrm{tree}} = \sqrt{\frac{-6m_R^2}{\lambda_R}} \qquad \omega_{3\mathrm{tree}} = -\sqrt{\frac{-6m_R^2}{\lambda_R}}. \tag{42}$$

If the φ field is stable, it must have lower energy than the plane-wave solution. Denote the plane-wave and soliton solutions by φ_1 and φ_2, respectively. The lowest-energy plane wave is

$$\varphi_1(x^-) = f_1^* \exp\left(\mathrm{i}\frac{\pi}{L}x^-\right) + f_1 \exp\left(-\mathrm{i}\frac{\pi}{L}x^-\right) \tag{43}$$

with a normalization constant $f_1 = 1/2\sqrt{L}$. It follows when $L \to \infty$ that

$$\int_{-L}^{+L} \mathrm{d}x^-\, \varphi_1^3(x^-) = \int_{-L}^{+L} \mathrm{d}x^-\, \varphi_1^4(x^-) = 0. \tag{44}$$

The constraint equation (3c) satisfied by $\varphi_1(x^-)$ is reduced to (41) with three solutions (42). This means the ϕ field has the same zero-mode field for φ_1 and φ_2. Thus, we can compare the field energies corresponding to φ_1 and φ_2. When $L \to \infty$, (7) gives

$$\begin{aligned} H_c|_{\varphi=\varphi_1} - H_c|_{\varphi=\varphi_2} &= \int_{-L}^{+L} \mathrm{d}x^-\, (\varphi_1^2 - \varphi_2^2)\left[\frac{\lambda_R}{4!}(\varphi_1^2 + \varphi_2^2) + \frac{m_R^2}{2} - \frac{\lambda_R}{4}\omega_{2\mathrm{tree}}^2\right] \\ &= -\int_{-L}^{+L} \mathrm{d}x^-\, \varphi_2^2\left[\frac{\lambda_R}{4!}\varphi_2^2 + \frac{m_R^2}{2} - \frac{\lambda_R}{4}\omega_{2\mathrm{tree}}^2\right]. \end{aligned} \tag{45}$$

A stable and localized φ_2 field must satisfy

$$H_c|_{\varphi=\varphi_1} > H_c|_{\varphi=\varphi_2}. \tag{46}$$

A strong restriction such that

$$\varphi_2^2(x^-) < -\frac{12m_R^2}{\lambda_R} + 6\omega_{2\mathrm{tree}}^2 \tag{47}$$

guarantees (46) absolutely. To seek an explicit soliton solution, we assume trial functions which are antisymmetric with respect to x^-,

$$\varphi_2(x^-) = \begin{cases} -2e^{-x^-}\left(e^{x^-}+e^{-x^-}\right)^{-1}\omega_{2\text{tree}} & x^- > 0 \\ 2e^{x^-}\left(e^{x^-}+e^{-x^-}\right)^{-1}\omega_{2\text{tree}} & x^- < 0 \\ 0 & x^- = 0 \end{cases} \tag{48}$$

or

$$\varphi_2(x^-) = \begin{cases} 2e^{-x^-}\left(e^{x^-}+e^{-x^-}\right)^{-1}\omega_{2\text{tree}} & x^- > 0 \\ -2e^{x^-}\left(e^{x^-}+e^{-x^-}\right)^{-1}\omega_{2\text{tree}} & x^- < 0 \\ 0 & x^- = 0 \end{cases} \tag{49}$$

When $x^- \to \pm\infty$, $\varphi_2(x^-)$ approaches zero exponentially. Thus, φ_2 is a localized field near $x^- = 0$ and is of a non-topological nature. Since the strong restriction (47) is satisfied by (48) or (49), the φ_2 field has lower energy than the plane wave φ_1 and is thus stable. The energy density of the φ_2 field with a finite energy is localized. Two non-topological soliton solutions for the φ_2 field satisfying the constraint equation have now been constructed. It is easy to see that the φ_2 in (48) corresponds to the kink of the ϕ field and that in (49) to the antikink of the ϕ field [34].

6. Numerical results and discussion

The effective potential has been derived up to the two-loop order. Infrared divergence does not occur as $k^+ \to 0$ since the field is massive. The ultraviolet divergence is cancelled by the counterterm. By numerically solving the equation $\mathrm{d}V(\omega)/\mathrm{d}\omega = 0$, the vacuum expectation value of the field ϕ can be obtained. Translation invariance of the theory [3, 12] ensures that the vacuum expectation value $\langle\phi\rangle$ is a constant. Generally, one solution satisfying $\mathrm{d}V(\omega)/\mathrm{d}\omega = 0$ is 0 and the other two have a sign difference.

The effective potentials in the tree, one-loop and two-loop approximations are plotted in figure 2 for a single pion field with mass m_π for which $m_R^2 = -m_\pi^2$ and $\lambda_R = 4m_\pi^2$.

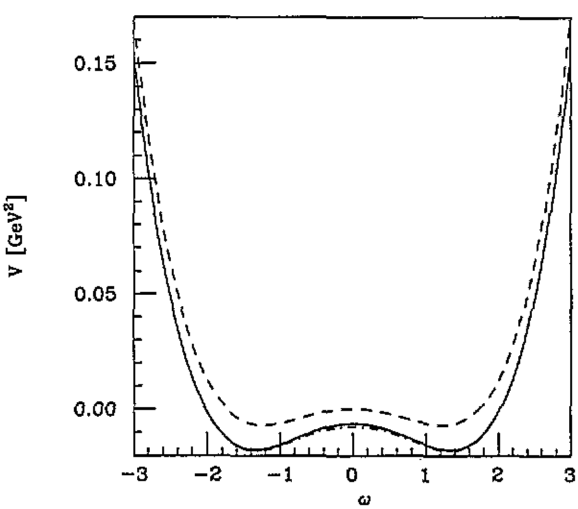

Figure 2. The effective potential as a function of ω is calculated with $m_0^2 = 0.1$ GeV^2, $\delta\bar{m}_1^2 = -(m_0^2 + m_\pi^2)$, $\delta m_2^2 = 0$, $\lambda_0 = 4m_\pi^2$, $\delta\lambda_1 = 0$ and $\delta\lambda_2 = 0$. The dashed (dash-dotted, solid) curve is the tree (one-loop, two-loop) approximation.

In the tree approximation the minimum of the effective potential is about $-0.007\ \mathrm{GeV}^2$ at $\omega \approx 1.15$. The one- and two-loop effective potentials give almost the same minimum value $-0.018\ \mathrm{GeV}^2$ at $\omega \approx 1.21$. The one-loop contribution $V_1(\omega)$ is important, while the two-loop correction $V_2(\omega)$ is small. The loop expansion (11) up to the two-loop order converges well in both the broken and symmetric phases. At $\omega = 0$, the three curves reach their local maximum. If the renormalized mass $m_{\mathrm{R}} = 0$, the maximum and the two minima coincide at $\omega = 0$. In this case, the effective potential is a parabola with its minimum located at $\omega = 0$, and $\langle\phi\rangle = 0$.

In the second-order phase transition, the vacuum expectation value of the ϕ field changes continuously through the critical point. Equation (25) is used to calculate critical coupling constants at the second-order phase transition which are, for example, $\lambda_{\mathrm{c}}'' = 52.46m_\pi^2$ for $\alpha = 40$ and $\gamma = 85$, and $\lambda_{\mathrm{c}}'' = 41.51m_\pi^2$ for $\alpha = 50$ and $\gamma = 95$. The results are consistent with the $40.0m_\pi^2$ in [20], the $59.5m_\pi^2$ in [10] and the values from $22m_\pi^2$ to $55m_\pi^2$ reported for the conventional equal-time field theory [35].

In the first-order phase transition, the vacuum expectation value has a discontinuity at the critical point. Equation (27) has two solutions in which one value is abolished by the restriction $m_0^2 > 0$. If the non-vanishing critical vacuum expectation value ϕ_{c} is known from experiments, the critical coupling constant at the first-order phase transition is obtained from (27). For instance, let $\phi_{\mathrm{c}} = 1$ with $\eta = 10$, then $\lambda_{\mathrm{c}}' = 48.76m_\pi^2$ with $\alpha = -37.41$ and $\gamma = 28.71$; let $\phi_{\mathrm{c}} = 1.2$ with $\eta = 6$, then $\lambda_{\mathrm{c}}' = 26.88m_\pi^2$ with $\alpha = -23.91$ and $\gamma = 23.22$.

We have seen that the effective potential can produce practical results provided the values of the finite and arbitrary quantities in m_{R} and λ_{R} are suitably selected. Nevertheless, it gives only a static description of the consequences of the phase transition since ω is a constant field.

7. Conclusions

The equivalence between the $\mathrm{d}V(\omega)/\mathrm{d}\omega = 0$ and the constraint $\theta_3 = 0$ is shown for the purpose of calculating the vacuum expectation value. The loop expansion given by Jackiw is applied to calculate the effective potential up to the two-loop order. The effective potential is shown to have a non-linear logarithmic structure. The one-loop contribution $V_1(\omega)$ is much bigger than the two-loop contribution $V_2(\omega)$. The criterion for the first- or second-order phase transition is obtained from the effective potential. Numerical results for the critical coupling constants indicate that the effective potential can give results consistent with other theories in light-cone quantization or conventional equal-time quantization. For the real scalar field we have given a static description of the spontaneous symmetry breaking in $1 + 1$ dimensions. When the zero and non-zero modes are assumed to be classical fields, the constraint equation is solved to obtain ω expressed in terms of φ in a power series in $\hbar$. In contrast to this infinitely distributed φ field in space, the static soliton solutions for the φ field have been constructed from the constraint equation. The existence of the non-topological soliton solution is a special result of the light-cone vacuum structure in comparison with the vacuum in the conventional equal-time theory. The investigation of classical field solutions in sections 4 and 5 has enlarged our understanding of the complexity and richness of the structure of the light-cone ϕ^4 field system.

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