Gluonic contributions in a four-fermion interaction model

W. Z. Jiang,[1,2,*] X. J. Qiu,[3] Z. Y. Zhu,[1,2] and Z. J. He[1,2]
[1]Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China
[2]Shanghai Institute of Nuclear Research, Chinese Academy of Sciences, Shanghai 201800, China
[3]Department of Physics, Shanghai University, Shanghai 201800, China

(Received 16 August 2001; published 20 December 2001)

The gap equation for the fermion in nuclear medium is obtained in a two-flavor gauged Nambu–Jona-Lasino (NJL) model using the Schwinger-Dyson (SD) equations. The gap equation is solved with a quenched truncation. Compared to the four-fermion interaction, the one-gluon-exchange interaction accounts for considerable contributions (about 15−50 %) to dynamically generated fermion mass. With incorporation of gluonic contributions into a scheme where there is only four-fermion interaction, the four-fermion coupling constant is made density dependent. Impacts of the density-dependent four-fermion (DDFF) coupling constants on quantities, such as the fermion mass and the chiral order parameter as well as masses of mesons (σ,π), are estimated. The DDFF coupling constants lead to less density dependence of hadron masses and the larger critical density of chiral symmetry restoration than those from the pure four-fermion interaction. The calculated quantities are somehow dependent on the confinement scale Λ_{QCD}. However, the range of Λ_{QCD} in the present parametrization can be determined by the saturation property of the gluonic contribution in the medium and it turns out quite small.

DOI: 10.1103/PhysRevC.65.015210 PACS number(s): 21.65.+f, 11.30.Rd

I. INTRODUCTION

Though the Nambu–Jona-Lasinio (NJL) model [1] cannot provide the mechanism of confinement and is nonrenormalizable, it is still a interesting and significant tool to investigate the nonperturbative low-energy physics of QCD [2] since the NJL model has properties of chiral symmetry and its spontaneous symmetry breaking as possessed in QCD. The NJL model is applied to particle physics in a wide range, such as $\pi^+-\pi^0$ mass difference [3], decay constants, scattering lengths [4], chiral soliton picture for baryons [5], etc.

The NJL model was originally proposed to describe the spontaneous symmetry breaking since the pion, as the Goldstone boson, can be derived dynamically in this model [1]. The chiral symmetry is expected to restore under external fields, such as the density or the temperature. Dynamical properties of the baryon and mesons are self-consistently constructed in the NJL model. In the past years, there were many works [6–11] to obtain the properties of the fermion and meson dynamics in the medium and thermal environment. The chiral symmetry restoration was investigated and the critical density or temperature was obtained in these works. In Ref. [12], the chiral symmetry breaking is studied under the external field, the gravitational field.

A convenient way to study the chiral symmetry breaking in the NJL model is through Schwinger-Dyson (SD) equations. Alternatively, the chiral symmetry breaking can be investigated by the standard renormalization group equations for coupling constants [13–15]. In this paper, we study the property of chiral symmetry breaking in nuclear medium via the SD equations.

One can show that the NJL model has some possible connections (such as symmetries) to QCD and may believe that the gluonic degree of freedom is integrated out [2] at the low-energy regime. However, the NJL model cannot be renormalized. In the past, some authors [16–18] took a reasonable step to extend the NJL model to the gauged one where the one-gluon-exchange interaction is included. Properties of chiral symmetry of QCD can be well simulated by the gauged NJL model. The gap equation of the gauged NJL model in the free space is obtained by using the SD equations in the Euclidean space, the phase structure is studied [19], and the renormalizability is discussed [20]. The solution to the gap equation indicates that there exists a large anomalous dimension [19–22] which implies that the situation of renormalization for the gauged NJL model can be improved. In Ref. [15], the guaged NJL model has been used to study the chiral symmetry property in the curved spacetime via the renormalization group equations. The top quark condensate, which relates directly to the large top quark mass in the composite Higgs model, has been investigated in the gauged NJL model, and it has been found that the presence of the gauged coupling is quite significant for the top quark mass [23,24]. Triggered by the gauged coupling contribution in the heavy-quark (high-energy scale) physics, one of objectives of this paper is to investigate the gluon-exchange contribution in the light flavor physics.

As to the point of the confinement of the model, simulating some behaviors of the confinement is possible because of the existence of gauge field in the gauged NJL model. Generally, the confinement is not really treated due to the complication in the gauged NJL model, whereas many of the low-energy static properties can be understood fairly well without the introduction of confinement. Hence we will not discuss deeply the details of the confinement.

In this paper, we investigate the gluonic contribution to the dynamically generated hadron masses and the chiral symmetry property in the gauged NJL model with flavor $N_f=2$ by virtue of the fermion gap equation. The fermion

*Corresponding author.

gap equation in nuclear medium will be derived by solving the SD equation for the fermion propagator where the four-fermion and one-gluon-exchange interactions are taken into account. The gluonic contribution in the gauged NJL model will be displayed by comparing to results without inclusion of gluonic interactions. At first, the gluonic contribution to the fermion mass will be estimated. The gluonic impacts on the quark condensate $\langle\bar{\psi}\psi\rangle_0$ and meson dynamics are then to be investigated. We will see that the gluonic effects in nuclear medium cannot be absorbed thoroughly in a four-fermion interaction theory by a constant coupling after integrating out the gluonic degree of freedom.

For clarity, we give three notations for models or framework used in the context. The model with the pure four-fermion interactions is mainly mentioned as the pure NJL, the model with both the four-fermion and gauge interactions is mainly called as the gauged NJL, and the framework with gauge interaction incorporated into four-fermion interactions is denoted as the density-dependent NJL (or, ρ-dependent NJL). We now give the arrangement for the paper. The gap equation in the nuclear medium for the fermion obtained from the SD equation is briefly investigated in Sec. II and other essential formulas are given. In Sec. III, numerical calculations are given and numerical results are discussed. In the final section, the summary and some discussions are given.

II. FERMION GAP EQUATION

Since the confinement is not specially considered for the static dynamical properties, it is more accurate to regard nuclear medium as quark matter. With the increasing density, nuclear medium will go closely to quark matter. The baryon is simply considered as a collective of three constituent quarks. Quarks are taken in the sense of constituent quarks here.

The NJL model on the quark level describes hadrons in terms of constituent quarks where the interactions are through the exchange of the quark-antiquark pairs. The one-boson-exchange interactions can find a dynamical origin in the NJL model. With introduction of the gluonic degree of freedom, the NJL model is modified to be the gauged one. The gauged NJL model can be expressed in terms of the following Lagrangian:

$$\mathcal{L}=\bar{\psi}(\not{D}+m_0)\psi+G[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2]+\sum_{\mu\nu\alpha}G^{\mu\nu}_{\alpha}G_{\mu\nu\alpha}. \tag{1}$$

Here ψ is the quark field, $G_{\mu\nu\alpha}$ is the gluon field strength with α the index of generator in the color space, and G is the bare four-fermion coupling constant.

The fermion gap equation in the free space [19] is obtained from the fermion propagator using SD equations, as also illustrated in Fig. 1. An analogous gap equation in nuclear medium is obtained. In order to derive the fermion gap equation in nuclear medium, some approximations are made. First, since the gap equation in nuclear matter is not covariant, the three-dimensional integral needs to be approximately transformed to the four-dimensional one using the Cauchy theorem for the analytic and numerical feasibility. Second, the gap quantity in the three-dimensional space is assumed to be only dependent on the three-dimensional momentum squared, similar to the four-dimensional case where the gap quantity is assumed to be dependent on the four-dimensional momentum squared. The reasonableness for the introduction of these approximations can be shown numerically in the following section. Since the gauged NJL model used here is nonrenormalizable, a quenched ultraviolet (uv) cutoff Λ is introduced to regularize the divergence. The detailed procedure for the deduction of the gap equation is presented in the Appendix. The gap equation in nuclear medium is written as

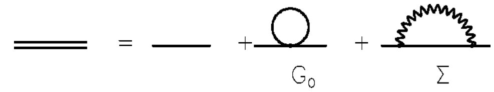

FIG. 1. Fermion propagator represented by the solution of the Schwinger-Dyson equation. Three parts on the right-hand side are the bare fermion propagator, the self-energy from the four-fermion, and one-gluon-exchange interactions, respectively.

$$\begin{aligned}B(x)=m_0&+\frac{g}{\Lambda^2}\int_{\Lambda_F^2}^{\Lambda^2}dy\frac{yB(y)}{y+B^2(y)}\\&+\int_{\Lambda_F^2}^{\Lambda^2}dy\frac{yB(y)}{y+B^2(y)}\left[\frac{\lambda(x)}{x}\theta(x-y)+\frac{\lambda(y)}{y}\theta(y-x)\right]\\&+\int_0^{\Lambda_F^2}dy\frac{yB(y)}{y+B^2(y)}\left[\frac{\lambda(y)}{y}-\frac{\lambda(x)}{x}\right]\theta(y-x).\end{aligned}\tag{2}$$

Here $x=q^2$ is a four-dimensional momentum covariant in the Euclidean space, $g=N_cN_fG\Lambda^2/4\pi^2$, and Λ_F, related to the fermi momentum, is determined as follows:

$$2\int_0^{k_F^2}dy\frac{y^{1/2}\tilde{B}}{(y+\tilde{B}^2)^{1/2}}=\int_0^{\Lambda_F^2}dy\frac{yB(y)}{y+B^2(y)},\tag{3}$$

where two approximations, made above, have been used. Since the integral limits for both integrals are the same topologically, $\tilde{B}$ is determined topologically by

$$\tilde{B}=B\left(y\frac{\Lambda_F^2}{k_F^2}\right).\tag{4}$$

For the gluonic interaction, it is renormalizable in principle. The reason to introduce the same uv cutoff to the gluonic interaction as to the four-fermion interaction is mainly due to the difficulty of the numerical treatment. Equation (2) is an integral equation, which is only capable of being solved numerically for the finite-momentum range. On the other hand, it is difficult to define the bare and renormalized quantities consistently in the nonrenormalizable Lagrangian. One may expect that the contribution from the high-momentum range can be approximately folded into the parameters (for instance, the four-fermion coupling G) describing the low-energy physics. Considering these factors, the gluonic interaction is approximately parametrized by the following simplistic running coupling constant $\lambda(q^2)$ [19], which may greatly suppress the integration beyond the uv cutoff:

$$\lambda(q^2)=\lambda_\mu(q^2)\,\theta(q^2-\mu^2)+\lambda_\mu\theta(\mu^2-q^2), \tag{5}$$

where

$$\lambda_\mu(q^2)=\frac{\lambda_\mu}{1+(\lambda_\mu/a)\ln(q^2/\mu^2)}=\frac{a}{(a/\lambda_\Lambda)+\ln(q^2/\Lambda^2)}=\frac{a}{\ln(q^2/\Lambda^2_{\rm QCD})} \tag{6}$$

with $a=9(n_c^2-1)/2n_c(11n_c-2n_f)$, $\lambda_\Lambda=\lambda(\Lambda^2)=a/\ln(\Lambda^2/\Lambda^2_{\rm QCD})$, and $\Lambda_{\rm QCD}$ the confinement scale whose value is specified in the following section. μ [equal to $\Lambda_{\rm QCD}\exp(a/2\lambda_\mu)$] plays the role of the infrared cutoff. The SD equation has the spontaneous-symmetry-breaking solution for $\lambda_\mu>\frac{1}{4}$ [25]. Theoretically, it is better to have a large value of λ_μ to count more confinement effect, while the calculated physical quantities seem almost independent of λ_μ at the region of large values, as will be specified in the following section. In the calculation, λ_μ is taken as 2.5.

In order to make the integral Eq. (2) solvable, we convert it to the differential equation

$$B''(x)=\frac{\left(\frac{\lambda(x)}{x}\right)''}{\left(\frac{\lambda(x)}{x}\right)'}B'(x)+\left(\frac{\lambda(x)}{x}\right)'\frac{xB(x)}{x+B^2(x)} \tag{7}$$

with the uv and infrared (ir) boundary conditions, respectively, as

$$B(z_\Lambda)+\frac{z_\Lambda}{1+z_\Lambda}\left(1+\frac{g}{a}z_\Lambda\right)B'(z_\Lambda)=m_0,$$

$$B'(z_{\Lambda_F})=0,$$

where $z=\ln(q^2/\Lambda^2_{\rm QCD})$. For the zero density, $\Lambda_F=0$. The fermion mass is defined by the normalization condition

$$B(q^2=M^2)=M. \tag{8}$$

The effective fermion mass M^* in nuclear medium is calculated at $q^2=M^2$.

With an increase in nuclear density, the contribution of the last integral to the fermion mass in Eq. (2) may exceed those of all other terms. Supposing an extreme case in which the nuclear medium is so dense that all terms except the last integral are negligible, the gap equation is, therefore, in the following simple form:

$$B(x)=m_0+\int_0^{\Lambda_F^2}dy\,\frac{yB(y)}{y+B^2(y)}\left[\frac{\lambda(y)}{y}-\frac{\lambda(x)}{x}\right]\theta(y-x). \tag{9}$$

Equation (9) has the same differential form as Eq. (7) and we are able to see that there is only a trivial void solution. As nuclear density exceeds the critical value, the chiral symmetry is restored completely. On the other hand, Eq. (9) explains the fact that even though the four-fermion contribution to the fermion mass vanishes, the gluonic interaction has nonvanishing contribution to the fermionic mass and related dynamical meson masses.

In order to investigate the fermion mass dependence on parameters (g,Λ), we will first give the numerical results at zero density as will be discussed in the following section. After this is done, the suitable g and Λ are determined to give the detailed numerical result for the dynamical fermion mass at various densities. Λ is determined by the fermion and meson (such as the pion) mass conditions in the case where the one-gluon-exchange interaction is not taken into account. We use an approximate relation $\Lambda=\sqrt{2}\Lambda_3$ where Λ_3 is the three-dimensional uv cutoff. Λ_3 can be determined via a four-fermion interacting model without gluonic terms.

The fermion and pion masses with only four-fermion interactions are determined in the following two formulas:

$$1-\frac{m_0}{M^*}=\frac{2G_0N_cN_f}{\pi^2}\int_{k_F}^{\Lambda_3}\frac{p^2dp}{E^*} \tag{10}$$

and

$$\frac{m_0}{M^*}-m_\pi^{*2}\frac{2G_0N_cN_f}{\pi^2}\int_{k_F}^{\Lambda_3}\frac{p^2dp}{E^*(4E^{*2}-m_\pi^{*2})}=0, \tag{11}$$

where k_F is the Fermi momentum, M^* and m_π^* are the effective fermion and pion masses in nuclear medium, respectively. $E^*=\sqrt{M^{*2}+p^2}$. G_0 is the four-fermion coupling when there is only the four-fermion interaction in the calculation. In the free space, the constituent mass M and m_π are set to be 313 and 138 MeV, respectively. With these two mass conditions G_0 and Λ_3 are uniquely determined. For completeness, we give the formula for σ mass in the following:

$$\frac{m_0}{M^*}-(m_\sigma^{*2}-4M^{*2})\frac{2G_0N_cN_f}{\pi^2}\int_{k_F}^{\Lambda_3}\frac{p^2dp}{E^*(4E^{*2}-m_\sigma^{*2})}=0. \tag{12}$$

Formulas for meson dynamics can also be found, for instance, in Refs. [7,8].

The chiral order parameter χ in gauged NJL is defined by the quark condensate

$$\chi=(\langle\bar{\psi}\psi\rangle_0)^{1/3}=N_c\int\frac{d^4p}{(2\pi)^4}{\rm tr}S(p)=\frac{N_c}{4\pi^2}\int^{\Lambda^2}dy\,\frac{yB(y)}{y+B^2(y)}=\frac{\Lambda^2N_c}{4\pi^2}\,\frac{B(z_\Lambda)-m_0}{g+\lambda_\Lambda}. \tag{13}$$

Without inclusion of gluonic interactions, the quark condensate is presented in the three-dimensional space as follows:

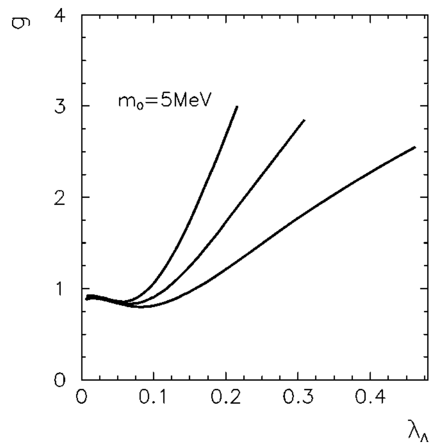

FIG. 2. Phase diagram of the fermion mass with parameters $(g,\lambda_\Lambda), M=313$, $m_0=5$ MeV. The different confinement scales $\Lambda_{\rm QCD}=120$, 160, and 200 MeV are for curves from bottom to top, respectively.

$$\langle\bar{\psi}\psi\rangle_0 = N_c\int\frac{d^4p}{(2\pi)^4}{\rm tr}S(p) = \frac{N_c}{\pi^2}\int_{k_F}^{\Lambda_3}p^2dp\frac{M^*}{E^*}. \quad (14)$$

III. NUMERICAL CALCULATION AND ANALYSIS

In this section, we do numerical calculations and give numerical analysis. After parameters are determined, quantities, such as the dynamically generated hadron masses, are calculated. At the same time, the gluonic contributions are discussed in detail and the property of chiral symmetry is investigated.

A. Parameter determination

Equation (7) can be solved by virtue of any two of three conditions: ir, uv, and normalization conditions. The relation of parameters g and Λ is worked out as ir and normalization conditions are used for solving the gap equation, and it turns out that the fermion mass is dependent on parameters (g,Λ). Figure 2 illustrates the constituent quark mass $M=313$ MeV with different $(g,\lambda_\Lambda(\Lambda))$'s. The given fermion mass in the free space corresponds to infinite number of (g,Λ) pairs, displaying various four-fermion and gluonic contributions to the fermion mass at zero density. The current fermion mass m_0 is set to be 5 MeV. The confinement scale $\Lambda_{\rm QCD}$ is an important quantity in the parametrization for the running gauge coupling and the numerical results are usually of confinement scale dependence. We will give the physical interpretation based on the numerical results worked out with the confinement scale $\Lambda_{\rm QCD}=120$, 160, and 200 MeV, while the dependence of physical quantities on $\Lambda_{\rm QCD}$ will be discussed at the end of the following subsection.

In models for four-fermion interactions, the cutoff Λ is needed to regularize the divergent integrations. Here the cutoff is determined by the constituent quark mass and the pion mass ($m_\pi=138$ MeV) conditions in pure NJL. The determined Λ is 973 MeV($\Lambda_3=688$ MeV). The same cutoff is used when the gluonic interaction is included, and the four-fermion coupling is thus decided by the relation of g and Λ. Only if the same cutoff is used for obtaining the same fermion mass, one can be aware of the quantitative gluonic contribution to the fermion mass in gauged NJL.

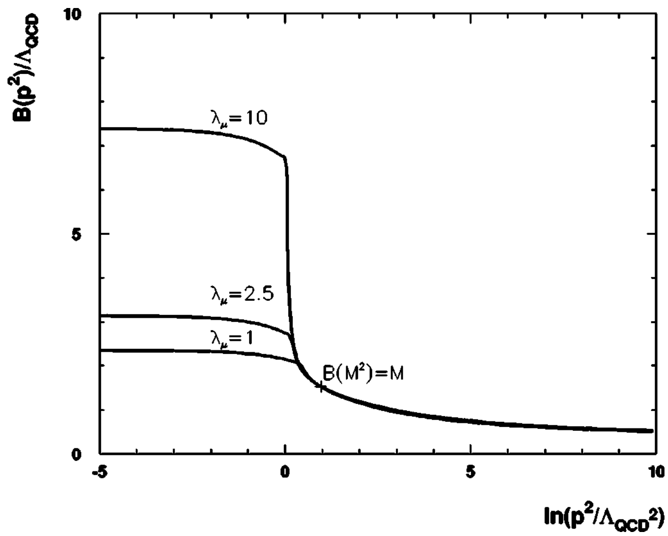

FIG. 3. $B(q^2)$ for different λ_μ's. The calculation is performed in the free space with $\Lambda_{\rm QCD}=200$, $M=313$, and $m_0=5$ MeV.

Before doing numerical calculations for physical observables, we need specify the parameter λ_μ. In the free space, the gap quantities $B(q^2)$ for different λ_μ's are plotted in Fig. 3, where we take the case $\Lambda_{\rm QCD}=200$ MeV as an example. As seen in the figure, the infrared value of the gauge coupling has a very small influence on the evolution of the gap quantity beyond the infrared-momentum range. Since the physical observables are actually specified or defined by the normalization condition which is used to solve the gap equation, the confining effect incorporated through the parameter λ_μ is negligible to the calculated physical quantities. For instance, the chiral order parameter χ, which can be evaluated by Eq. (13) at Λ, has the distinction much less than 1% for different λ_μ's. As long as the spontaneous-symmetry-breaking solution of the SD equation is generated ($\lambda_\mu>\frac{1}{4}$), the calculated physical quantities are almost independent of λ_μ for a large range. In nuclear matter, numerical calculations indicates that calculated physical quantities are also almost independent of λ_μ, and that is not specially illustrated below.

B. Fermion mass and four-fermion coupling in medium

The definition for the nuclear density is given as $\rho=N_cN_fk_F^3/3\pi^2$ with the normal density $\rho_0=0.16$ fm^{-3}. The fermion mass decreases with increasing density as shown in Fig. 4 and $M^*\to m_0$ at high densities. The results with various $\Lambda_{\rm QCD}$'s indicate that the smaller $\Lambda_{\rm QCD}$ corresponds to the larger critical density while this can be seen in Fig. 4 where three curves correspond, respectively, to $\Lambda_{\rm QCD}=120$, 160, and 200 MeV. To obtain the same fermion mass under same uv cutoff in the free space, one can use different combination of the four-fermion coupling g and the $\Lambda_{\rm QCD}$ that scales the gluonic contribution. The effective mass is dependent on $\Lambda_{\rm QCD}$. Here $g=0.892$, 0.983, and 1.049 determined in the

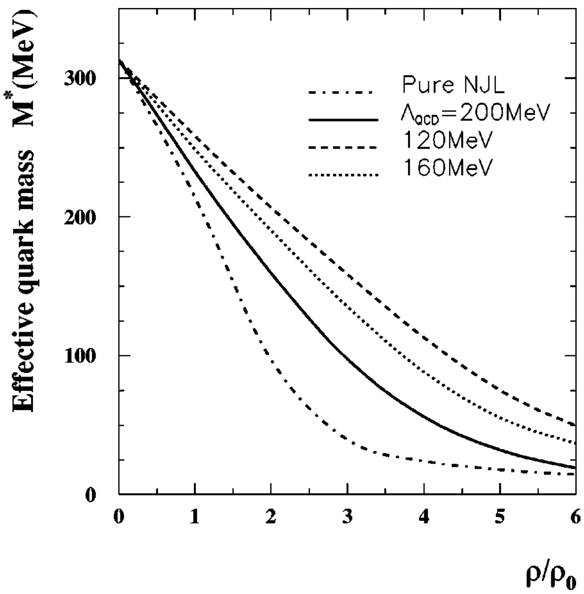

FIG. 4. Effective fermion masses in nuclear medium. $\rho_0 = 0.16\ \mathrm{fm}^{-3}$.

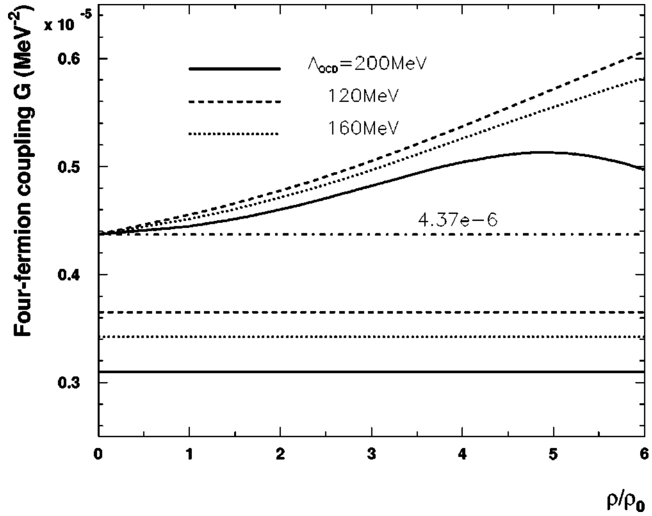

FIG. 5. Density dependence of the four-fermion coupling. The short dot-dashed line is for G_0 in pure NJL. Other straight lines stand for density-independent four-fermion couplings in gauged NJL, while curves are for four-fermion couplings in ρ-dependent NJL with Λ_{QCD} denoted in the figure.

free space correspond, respectively, to $\Lambda_{\mathrm{QCD}}=200$, 160, and 120 MeV, and that shows the larger Λ_{QCD} gives the larger gluonic contribution to the fermion mass in the free space. In a framework of the four-fermion interaction plus the one-gluon-exchange interaction, the pure four-fermion interaction can be obtained as the running gauge coupling vanishes, as seen in Fig. 2. Meanwhile, it is found in Fig. 2 that the gauged NJL interaction can go without much change of the four-fermion coupling to the pure four-fermion interaction from some large Λ (small λ_Λ) by simply integrating out the gluonic interaction up to very high uv cutoff. However, in the practical calculation, the uv cutoff, which is usually below 1 GeV determined by physical quantities, is used for regularizing the divergent integrals. Therefore, it is possibly but not absolutely right that reasonably large gluonic contribution in gauged NJL can make some results approach to those in pure NJL. These explain why the effective fermion mass with larger Λ_{QCD} is closer to that in pure NJL. Meanwhile, the consistency between the numerical results obtained here and the two approximations made at the beginning to derive the gap equation indicate that the approximations are reasonable. However, this is not to say that the results of gauged NJL can be reproduced by the pure NJL, as we will see in Sec. III C that a better chiral order parameter can be obtained only if the gauge interaction is included.

Gluonic contribution to the fermion mass is scaled through comparison of the four-fermion couplings with and without gluonic interactions under the same uv cutoff condition. After parameters are decided, the effective fermion mass in gauged NJL is obtained by solving the gap equation in nuclear medium. For the NJL model, the four-fermion coupling is re-adjusted to get the same effective fermion mass as in gauged NJL at different densities. In the free space, gluonic interaction accounts for the dynamical origin of the fermion mass by a factor from about 15% to 30%, depending on the confinement scale. Figure 5 illustrates the four-fermion coupling with inclusion of gluonic interactions with different Λ_{QCD}'s. At zero density, the value of G on curves (not on straight lines) is for the pure NJL. It shows that by incorporation of gluonic interactions the four-fermion coupling $G_0(\rho)$ is strongly density dependent.

More explicitly, we plot the extracted gluonic contribution in Fig. 6. The extracted gluonic contribution factor Σ_g is defined by

$$\Sigma_g = \frac{G_0(\rho) - G}{G_0} . \tag{15}$$

The gluonic contribution to the effective fermion mass could increase from about 20% to as high 50% in medium at high densities. As seen in Figs. 5 or 6, $G_0(\rho)$ increases monoto-

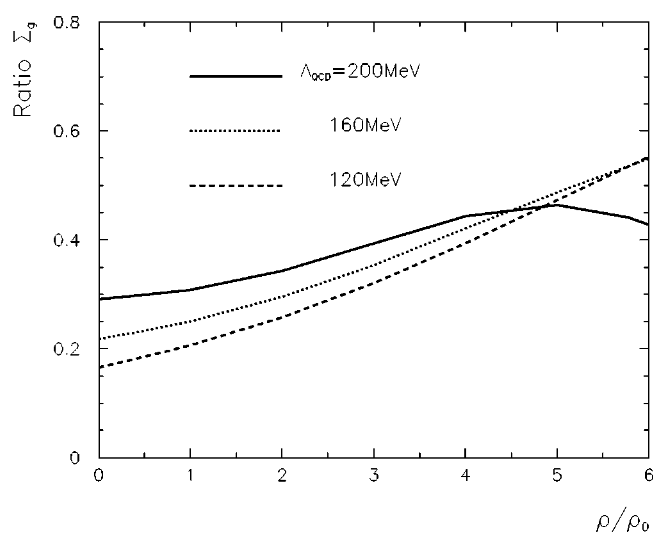

FIG. 6. Extracted gluonic contribution in nuclear medium.

TABLE I. Chiral order parameter χ for different confinement scale Λ_{QCD}.

Λ_{QCD} (MeV)	120	160	200	240
χ (MeV)	247.3	243.1	236.1	219.5

nously with increasing density for small Λ_{QCD}'s. Increasing Λ_{QCD}, $G_0(\rho)$ may have the maximum along the density axis. By incorporation of the gluonic contribution, the density-dependent four-fermion (DDFF) coupling is induced naturally, and that leads to a density-dependent NJL.

By now, we need to make a discussion for the dependence of relevant quantities on Λ_{QCD}. Λ_{QCD} has a big error including theoretical uncertainties, say, roughly from 100 to 500 MeV, depending on parametrization scheme of the running gauge coupling constant [26,27]. For integrals of the four-fermion interaction in nuclear medium, the Fermi momentum plays the role as an ir cutoff, whereas it is not a pure ir cutoff as shown in Eq. (2). At densities not very high, the four-fermion contribution can decrease faster than the gluonic one in the gap equation for some Λ_{QCD}'s with the increasing density. The gluonic contribution must reduce faster than the four-fermion one at high densities where the running gauge coupling at short range plays the role. Therefore, there must be a turning point from increase to decrease on the curve for $G_0(\rho)$. This provides a constraint on the uncertainties of Λ_{QCD}. Using this constraint, we see from Fig. 6 that the Λ_{QCD} cannot be very small, say, it cannot be much lower than 200 MeV. On the other hand, the gluonic contribution decreases with the increasing density for larger Λ_{QCD} (for instance, $\Lambda_{QCD}\geqslant 240$ MeV). The gluonic contribution should reach its maximum in medium, which is actually a saturation property of the gluonic contribution. So the reasonable range for Λ_{QCD} should be between the places where the gluonic contribution decreases or increases monotonously with the increasing density. From present parameters, numerical calculations indicate that Λ_{QCD} ranges roughly from 190 to 240 MeV, which is a rather small range.

C. Chiral order parameter with gluonic contribution

For different Λ_{QCD}'s, we have calculated the corresponding chiral order parameters at zero density as given in Table I. As shown in Table I, χ's with a large scope of Λ_{QCD} in gauged NJL are within the error bar predicted by the QCD sum rules [28] (225 ± 35 MeV) or in [29] (225 ± 25 MeV). This is a good indication that the gauged NJL can simulate the low-energy physics of QCD quite well through choosing the suitable confinement scales. The chiral order parameter is much less sensitive to Λ_{QCD} than the effective fermion mass. Compared to the chiral order parameters in [28,29], it implies that a reasonably larger Λ_{QCD} is preferred to obtain a reasonably smaller χ. In general, the chiral symmetry property can be well simulated in gauged NJL. It is necessary to note that χ is 260.1 MeV for the pure NJL. The gauged NJL model gives a better chiral order parameter.

Besides the four-fermion coupling, the chiral order parameter is also effectively modified through incorporation of gluonic contribution. Figure 7 plots the chiral order parameter with respect to the density. The two curves that are calculated with density-independent and DDFF coupling constants indicate that gluonic interactions play their important roles in the chiral order parameter. With inclusion of the effective gluonic contribution, the chiral order parameter approaches to that of the gauged NJL. The gluonic interactions can effectively modify the behavior of chiral symmetry restoration, represented by the chiral order parameter.

D. Meson (σ,π) masses with gluonic contribution

The σ and π masses in ρ-dependent NJL can be calculated by Eqs. (11) and (12) but with DDFF coupling constants. As an example of application for these DDFF coupling constants in the density-dependent NJL, meson masses are calculated from the case of $\Lambda_{QCD}=200$ MeV. Here we investigate the influence of DDFF coupling constants on meson masses and the property of chiral symmetry.

The DDFF coupling explicitly modifies the density-dependent behavior of effective meson masses. The less density-dependent meson masses are given by the ρ-dependent NJL than by the pure NJL, as is shown in Fig. 8. The critical density of chiral restoration is also effectively modified by the DDFF coupling. It may have two kinds of criteria to determine the critical point. One is that $m_\pi^*\to m_\sigma^*$ is used for criterion, the other is that the shift point where the

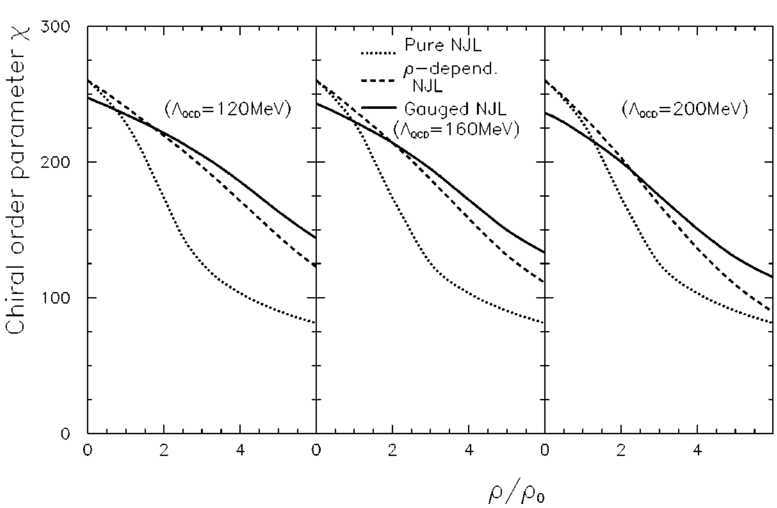

FIG. 7. Chiral order parameter χ in nuclear medium.

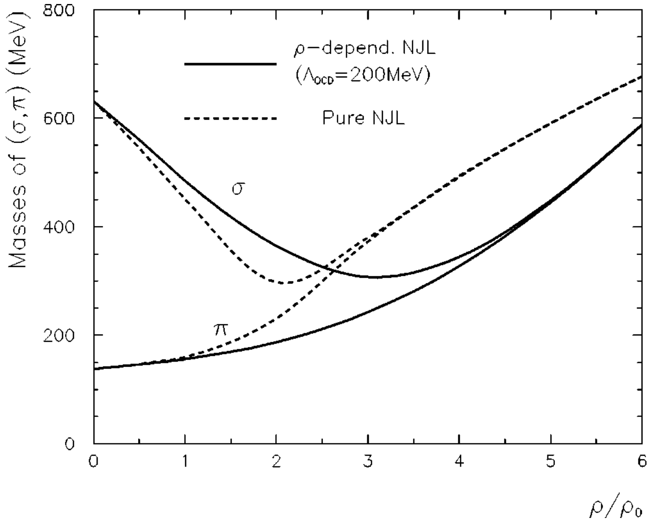

FIG. 8. Masses of chiral partner (σ,π) in nuclear medium.

sigma mass goes from decrease to increase with increasing density is regarded as the critical point. No matter which criterion is used, the larger critical density is given by the ρ-dependent NJL and the difference of the critical density between the ρ-dependent NJL and pure NJL has about more than $1\rho_0$, as shown in Fig. 8. Though this difference is different numerically for different Λ_{QCD}, either the critical density or dynamical masses of mesons are effectively modified without exception by the gluonic contribution. Compared to the ρ-dependent NJL, the pure NJL model may overestimate the density-dependent behavior of effective meson masses. In short, the gluonic contribution has non-negligible influence on meson dynamics and chiral symmetry property in medium.

IV. SUMMARIES

Based on the derived gap equation in nuclear medium, we have investigated a variety of quantities: fermion mass, DDFF coupling constants, chiral order parameter, and so forth. The gluonic impacts are displayed through comparing to results in pure NJL. Due to inclusion of gluonic interaction, the chiral order parameter is improved and theoretical predictions of the fermion and meson masses are effectively modified. The chiral order parameter χ predicted here is consistent with the one by the QCD sum rules. The gluonic contribution is essentially important to explain the dynamical origin of the fermion mass in the four-fermion interaction models either in vacuum or in medium. The gluonic interaction can explain the dynamic origin of fermion mass by a factor of about 15–30 % in the free space, and the maximum of the gluonic contribution reaches in medium. The DDFF coupling is obtained as the gluonic contribution is mapped into the four-fermion interaction framework. One implication from the DDFF coupling is that the gluonic contribution cannot be simply integrated out with its contribution absorbed by the constant four-fermion coupling in pure NJL.

The significance of the DDFF coupling is shown in obtaining the effective fermion mass, the chiral order parameter, and the meson masses. With inclusion of gluonic contribution, it has less density dependence for the fermion and meson masses and the chiral order parameter, meanwhile the critical density of chiral symmetry restoration becomes larger. Due to the importance of gluonic contribution, we may expect more applications for the ρ-dependent NJL in a wide range of hadronic physics later on.

Besides simplistic treatments of parametrization for the gluonic degree of freedom, we have used some approximations to derive the gap equation in nuclear medium. With these approximations, the three-dimensional integral has been numerically transformed to the four-dimensional one by using the Cauchy integration. At the same time, the quenched truncation in the gap equation is used to carry out numerical results since the present four-fermion interaction model is nonrenormalizable. The quenched uv cutoff has been also introduced to the gluonic interaction in gauged NJL and it would be necessary to take more considerations (including renormalization) to deal with the gluonic interaction later on. These approximations and simplistic treatments could induce some error. However, the present theoretical predictions are reasonable and consistent with the analytical analysis and empirical data as well. In addition, the present numerical results are primitive since the exact constituent quark mass is not known though the relation $M_N = 3M$ is used.

In summary, we have obtained the gap equation in nuclear medium using the SD equations. The gluonic contribution turns out very significant for the DDFF coupling constants, hadron masses and the property of chiral symmetry. The error including theoretical uncertainties of the confinement scale is greatly reduced according to the saturation property of the gluonic contribution in nuclear medium using the SD equation-based approach. The range of Λ_{QCD} is about 50 MeV in the present parametrization of the running coupling constants.

APPENDIX: DERIVATION OF THE GAP EQUATION

The fermion gap equation can be derived by solving the SD equations for the fermion propagator. In the Hartree approximation, these SD equations in the free space are written as

$$S_F(q)^{-1} = S_F^0(q)^{-1} + \Sigma(q) - iG\langle\bar{\psi}\psi\rangle \tag{A1}$$

and

$$S_F^0(q)^{-1} = \gamma_\mu q^\mu - m_0, \tag{A2}$$

$$\Sigma(q) = -i\int \frac{d^4p}{(2\pi)^4}\gamma_\mu S_F(p)\Gamma_\nu D^{\mu\nu}(q-p), \tag{A3}$$

$$\langle\bar{\psi}\psi\rangle = N_c\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}[S_F^0(p)], \tag{A4}$$

$$D_{\mu\nu}(p) = \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)\frac{d(p^2)}{p^2} - \frac{\alpha}{p^2}\frac{p_\mu p_\nu}{p^2}, \tag{A5}$$

where d is the coupling constant, α is the gauge parameter, and Γ_ν is the interaction kernel, which is replaced by γ_ν in the actual calculation. The following self-consistent interaction fermion propagator is assumed:

$$S_F(q)^{-1}=A\gamma_\mu q^\mu - B, \tag{A6}$$

where the gap quantities A and B can be determined by Eq. (A1). A can be proved to be unity by taking the Landau gauge [18]. B can be calculated from the following equation:

$$B=m_0+\frac{\mathrm{tr}\Sigma(q)}{trI}+G\langle\bar{\psi}\psi\rangle \tag{A7}$$

with I the unit matrix. Here we are not to go into details of the deduction that can be referred to [18]. Through the straightforward calculation, we can arrive at the gap equation in the free space [19] as follows:

$$B(x)=m_0+\frac{g}{\Lambda^2}\int_0^{\Lambda^2}dy\,\frac{yB(y)}{y+B^2(y)} + \int_0^{\Lambda^2}dy\,\frac{yB(y)}{y+B^2(y)}\left[\frac{\lambda(x)}{x}\,\theta(x-y)+\frac{\lambda(y)}{y}\,\theta(y-x)\right]. \tag{A8}$$

In nuclear medium, the fermion propagator has two terms one of which is the same as the one in the free space and the other gives the medium modification

$$S(p)=(\gamma^\mu p_\mu+B)\left[\frac{1}{p^2-B^2+i\epsilon}+\frac{i\pi}{E(p)} \times\delta(p_0-E(p))\theta(k_F-|\mathbf{p}|)\right] \tag{A9}$$

with $E(p)=\sqrt{\mathbf{p}^2+B^2}$. Thus the fermion condensate $\langle\bar{\psi}\psi\rangle$ is

$$\langle\bar{\psi}\psi\rangle=N_c\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}[S(p)]=\frac{2N_c}{(2\pi)^2}\int_{p_F^2}^{\Lambda_3^2}dy\frac{y^{1/2}\tilde{B}}{(y+\tilde{B}^2)^{1/2}}, \tag{A10}$$

where $\tilde{B}$ is defined in Eq. (4), and the Cauchy theorem is used for obtaining the second equality as the four-dimensional integral is converted into three dimensional one. Using the Cauchy theorem inversly, Eq. (A10) is converted into the four-dimensional form

$$\langle\bar{\psi}\psi\rangle=\frac{N_c}{(2\pi)^2}\int_{\Lambda_F^2}^{\Lambda^2}dy\,\frac{yB}{y+B^2}. \tag{A11}$$

Since the formulas are actually noncovariant in nuclear medium, the approximation is introduced when the conversion between the covariant (four-dimensional) and three-dimensional forms is made to obtain the above equation and the second equality of Eq. (A10). The infrared cutoff Λ_F is determined by keeping the equality of two integrals at the both sides of Eq. (3).

For the gluonic part in nuclear medium, it is divided into two portions, in a manner similar to the four-fermion part. One portion is the same as that in the free space, and the other medium-related portion is calculated as follows:

$$\begin{aligned}
&-\int\frac{d^4p}{\pi^2}\,\frac{\pi B}{(\mathbf{p}^2+B^2)^{1/2}}\,\delta(p_0-E(p))\theta(k_F-|\mathbf{p}|)\\
&\quad\times\left[\frac{\lambda(q^2)}{q^2}\,\theta(q^2-p^2)+\frac{\lambda(p^2)}{p^2}\,\theta(p^2-q^2)\right]\\
&=-2\int^{k_F^2}d\mathbf{p}^2(\mathbf{p}^2)^{1/2}\frac{\tilde{B}}{(\mathbf{p}^2+\tilde{B}^2)^{1/2}}\\
&\quad\times\left[\frac{\lambda(q^2)}{q^2}\,\theta(q^2+\tilde{B}^2)-\frac{\lambda(-\tilde{B}^2)}{\tilde{B}^2}\,\theta(-q^2-\tilde{B}^2)\right]\\
&=-2\int^{k_F^2}dy\,\frac{y^{1/2}\tilde{B}}{(y+\tilde{B}^2)^{1/2}}\,\frac{\lambda(x)}{x}\\
&=-\int^{\Lambda_F^2}dy\,\frac{yB}{(y+B^2)}\,\frac{\lambda(x)}{x}
\end{aligned} \tag{A12}$$

with $x=q^2$.

Adding these medium-related terms to Eq. (A8), the gap equation in nuclear medium is

$$B(x)=m_0+\frac{g}{\Lambda^2}\int_{\Lambda_F^2}^{\Lambda^2}dy\frac{yB(y)}{y+B^2(y)} + \int_0^{\Lambda^2}dy\,\frac{yB(y)}{y+B^2(y)}\left[\frac{\lambda(x)}{x}\,\theta(x-y)+\frac{\lambda(y)}{y}\,\theta(y-x)\right] - \int_0^{\Lambda_F^2}dy\,\frac{yB(y)}{y+B^2(y)}\,\frac{\lambda(x)}{x}. \tag{A13}$$

The contraction for the last two integrals of the right-hand side of Eq. (A13) leads to Eq. (2) directly.

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[2] J. Bijnens, Phys. Rep. **265**, 369 (1996).
[3] J. Bijnens and E. de Rafael, Phys. Lett. B **273**, 483 (1991).
[4] T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994).
[5] R. Alkofer, H. Reinhardt, and H. Weigel, Phys. Rep. **265**, 139 (1996).
[6] V. Bernard, Ulf-G. Meissner, and I. Zahed, Phys. Rev. D **36**, 818 (1987).

[7] V. Bernard and Ulf-G. Meissner, Nucl. Phys. **A489**, 647 (1988).
[8] E.M. Henley and H. Müther, Nucl. Phys. **A513**, 667 (1990).
[9] Chr.V. Christov, E. Ruiz Arriola, and K. Goeke, Nucl. Phys. **A510**, 689 (1990).
[10] P. Zhuang, J. Hüfner, and S.P. Klevansky, Nucl. Phys. **A576**, 525 (1994).
[11] J. Cugnon, M. Jaminon, and B. Van den Bossche, Nucl. Phys. **A598**, 515 (1996).
[12] S. Leseduarte and S.D. Odintsov, Phys. Rev. D **49**, 5551 (1994).
[13] W. Bardeen, C. Hill, and M. Lindner, Phys. Rev. D **41**, 1647 (1990).
[14] M. Harada, Y. Kikukawa, T. Kugo, and H. Nakano, Prog. Theor. Phys. **92**, 1161 (1994).
[15] B. Geyer and S.D. Odintsov, Phys. Rev. D **53**, 7321 (1996).
[16] C.N. Leung, S.T. Love, and W.A. Bardeen, Nucl. Phys. **B273**, 649 (1986); W.A. Bardeen, C.N. Leung, and S.T. Love, *ibid.* **B323**, 493 (1989).
[17] T. Appelquist, M. Soldate, T. Takeuchi, and L.C.R. Wijewardhara, in *Proceedings of the 12th Johns Hopkins Workshop on Currect Problems in Particle Theory*, edited by G. Domokos and S. Kovesi-Domokos (World Scientific, Singapore, 1988).
[18] K-I. Kondo, H. Mino, and K. Yamawaki, Phys. Rev. D **39**, 2430 (1989).
[19] K-I. Kondo, Susumu Shuto, and K. Yamawaki, Mod. Phys. Lett. A **6**, 3385 (1991).
[20] K-I. Kondo, M. Tanabashi, and K. Yamawaki, Mod. Phys. Lett. A **8**, 2859 (1993).
[21] V.A. Miransky and K. Yamawaki, Mod. Phys. Lett. A **4**, 129 (1989).
[22] V.A. Miransky, T. Nonoyama, and K. Yamawaki, Mod. Phys. Lett. A **4**, 1409 (1989).
[23] V.A. Miransky, M. Tanabashi, and K. Yamawaki, Phys. Lett. B **221**, 177 (1989).
[24] F. King and S.H. Mannan, Phys. Lett. B **241**, 249 (1990).
[25] K. Higashijima, Phys. Rev. D **29**, 1228 (1984).
[26] W.J. Marciano, Phys. Rev. D **29**, 580 (1984).
[27] I. Hinchliffe, Phys. Rev. D **54**, 77 (1996).
[28] L.J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[29] G.Q. Li and C.M. Ko, Phys. Lett. B **338**, 118 (1994).